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Brief paper

Optimal switching instants for a switched-capacitor DC/DC power converter☆

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ABSTRACT

We consider a switched-capacitor DC/DC power converter with variable switching instants. The determination of optimal switching instants giving low output ripple and strong load regulation is posed as a non-smooth dynamic optimization problem. By introducing a set of auxiliary differential equations and applying a time-scaling transformation, we formulate an equivalent optimization problem with semi-infinite constraints. Existing algorithms can be applied to solve this smooth semi-infinite optimization problem. The existence of an optimal solution is also established. For illustration, the optimal switching instants for a practical switched-capacitor DC/DC power converter are determined using this approach.

1. Introduction

DC/DC power converters are used in mobile electronic systems such as laptop computers and cellular phones to generate different DC voltages from a single battery source. Over the past two decades, many modern DC/DC power converters have been developed that can be realized primarily using capacitors and switches (see, for example, Chung and Mok (1999) or Chung, Chow, Hui, and Lee (2000) and the references cited therein). Such power converters are called *switched-capacitor DC/DC power converters*. Free of bulky inductive elements, they are ideal for small-size applications requiring low electromagnetic interference and high power density.

The capacitors in a switched-capacitor DC/DC power converter are used to store and supply energy. The circuit topology and, in particular, the function of each capacitor changes according to the switch configuration. For each switch configuration, some of the capacitors act as the power supply and deliver energy to the load; the remainder are charged by the input source. The converter operates by switching between the different topologies so that the role of each capacitor is changed regularly. More specifically, when a topology switch occurs, those capacitors that were previously discharging energy to the load begin to charge up, while those that were previously charging start to release energy as output voltage. For more detailed information, the reader can consult Ioinovici (2001) and the references cited therein.

Ideally, any DC/DC power converter should supply a steady voltage to the attached appliance. However, the switching mechanism inherent in the operation of a switched-mode power converter induces a ripple in the output voltage. Hence, although the ripple may be reduced by increasing the switching frequency, it is impossible to eliminate it entirely. On the other hand, topology switches are accompanied by an energy loss, and so excessive switching should be avoided (see Arntzen and Maksimović (1998)). Furthermore, the input voltage and load resistance influence the converter output through the circuit dynamics. This influence should be minor so that uncertainties in the input and changes to the load do not cause large variations in the output voltage.

A switched-capacitor DC/DC power converter can be controlled by varying the duty cycle – that is, the time spent in each topology – using a pulse-width modulation technique. In view of the previous discussion, an ideal control scheme would achieve the following two objectives: (i) minimize the output voltage ripple; and (ii) ensure output voltage regulation in the presence of uncertainties.

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Many different feedback control methodologies for achieving one, or both, of these objectives have been proposed in the literature. See, for example, Choi, Lim, and Choi (2001), Garofalo, Marino, Scala, and Vasca (1994), Khayatian and Taylor (1994), or Leung, Tam, and Li (1993), and the references cited therein. The majority of these methods are based on a linear time invariant approximate model of the switched-capacitor DC/DC power converter. However, since its governing dynamics change at the switching instants, a switched-mode power converter actually constitutes a highly non-linear and time-varying dynamical system. Hence, the performance of these existing control schemes can only be guaranteed under a small signal assumption.

In contrast, the problem of determining optimal switching instants *a priori* has received little attention in the literature. In Ho, Ling, Liu, Tam, and Teo (2008), a novel method for the offline computation of these switching instants was proposed. Specifically, the problem was formulated as a dynamic optimization problem, where the switching instants are chosen to minimize a cost function subject to a dynamic model of the power converter. This problem can be solved using existing optimization software such as MISER (see Jennings, Fisher, Teo, and Goh (2004)). The switching instants obtained can then be used to operate the power converter.

Unlike previous control schemes, this approach avoids the use of averaging and linearization; instead, a more accurate switched system dynamic model of the power converter is used. The time-varying and non-linear nature of a switched-mode power converter is therefore explicitly taken into account in the offline formulation of an optimal switching regime. Incidentally, the optimization and control of switched systems has been an active research area over the past decade, and we direct the interested reader to Bengea and DeCarlo (2005), Seatzu, Corona, Giua, and Bemporad (2006), and Xu and Antsaklis (2004) for information on some recent developments.

The cost function used by Ho et al. (2008) contains terms that penalize *both* the output voltage ripple and the output sensitivity. Hence, objectives (i) and (ii) above are simultaneously considered in the determination of an optimal switching scheme. Calculating these output sensitivity terms, however, is a complicated task involving matrix inversion, eigenvalue computation, and a formula consisting of five nested summations. Therefore, computing the cost function and, in particular, its gradient, is highly involved. Nevertheless, this computation is necessary to solve the optimization problem effectively. Furthermore, the eigenvalues of the system coefficient matrices need to be derived analytically as functions of the load resistance. Such analytical expressions are only possible if the system coefficient matrices have dimension less than or equal to four. Thus, the method proposed in Ho et al. (2008) is only applicable to problems with small dimension. This is a serious restriction, and hence there is an urgent need to develop a more efficient method that can be applied to the large-scale problems encountered in practice.

With this motivation, in this paper we formulate the determination of optimal switching instants as a different optimization problem to that discussed in Ho et al. (2008). We penalize output voltage ripple over the entire time horizon, and not separately over each topology. Hence, the switching loss penalty terms introduced by Ho et al. (2008) become redundant. Furthermore, a novel method is developed to calculate the output sensitivity terms via an auxiliary system of differential equations. This auxiliary system can be solved simultaneously with the state system using any standard differential equation solver. Thus, the computation of the sensitivity terms is a simple and straightforward exercise, in contrast with the arduous task required in Ho et al. (2008). We also establish the existence of an optimal solution in Section 5 before applying our method to a practical example in Section 6.

2. Problem formulation

Consider a switched-capacitor DC/DC power converter containing m capacitors. Suppose that during the time horizon $[0, T]$, the converter switches topology n times. In other words, it cycles through $n+1$ different circuit topologies in each switching period. Since physical considerations limit the maximum rate of switching, there is a minimum time duration $\rho > 0$ that must be spent in each topology. On this basis, define

$$\Gamma := \left\{ \boldsymbol{\tau} \in \mathbb{R}^n : \tau_i + \rho \leq \tau_{i+1}, i = 0, \ldots, n \right\},$$

where $\tau_0 = 0$ and $\tau_{n+1} = T$. The power converter can be operated using the components of a given $\boldsymbol{\tau} \in \Gamma$ as the topology switching instants. Accordingly, any $\boldsymbol{\tau} \in \Gamma$ is referred to as a feasible vector of switching instants.

Now, for each $i = 1, \ldots, m$, let $x_i(t) \in \mathbb{R}$ denote the voltage across the ith capacitor at time t. Furthermore, let $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_n]^{\mathrm{T}} \in \Gamma$. Topology switches are accompanied by a voltage loss from the capacitors in the converter. We assume that this voltage leak can be expressed as a given function of the voltage across the capacitors immediately before the switch. Accordingly, the state voltage $\mathbf{x}(t) = [x_1(t), \ldots, x_m(t)]^{\mathrm{T}} \in \mathbb{R}^m$ experiences a jump at each switching instant:

$$\mathbf{x}(\tau_i^+) = \mathbf{x}(\tau_i^-) + \boldsymbol{\phi}^i(\mathbf{x}(\tau_i^-)), \quad i = 1, \ldots, n, \tag{1}$$

where the negative and positive superscripts denote the limit from the left and right, respectively, and $\boldsymbol{\phi}^i : \mathbb{R}^m \to \mathbb{R}^m$, $i = 1, \ldots, n$, are given continuously differentiable functions.

During the ith topology, $i = 1, \ldots, n+1$, the state voltage is governed by a linear time invariant dynamical system as follows:

$$\dot{\mathbf{x}}(t) = A^i\mathbf{x}(t) + B^i\boldsymbol{\sigma}, \quad t \in (\tau_{i-1}, \tau_i), \tag{2}$$

where $\boldsymbol{\sigma} = [\sigma_1, \ldots, \sigma_r]^{\mathrm{T}} \in \mathbb{R}^r$ is the DC input voltage, $R_L \in \mathbb{R}$ is the given load resistance and, for each $i = 1, \ldots, n+1$, $A^i := A^i(R_L) : \mathbb{R} \to \mathbb{R}^{m\times m}$ and $B^i := B^i(R_L) : \mathbb{R} \to \mathbb{R}^{m\times r}$ are given matrix-valued functions of the load resistance. We assume that each of these functions is continuously differentiable.

The initial condition for the dynamics (2) is:

$$\mathbf{x}(0) = \mathbf{x}(0^+) = \mathbf{x}^{\mathbf{0}}, \tag{3}$$

where $\mathbf{x}^{\mathbf{0}} \in \mathbb{R}^m$ is the initial voltage across the capacitors.

The output voltage delivered by the converter during the ith topology, $i = 1, \ldots, n+1$, is given by

$$y(t) = C^i\mathbf{x}(t) + D^i\boldsymbol{\sigma}, \quad t \in [\tau_{i-1}, \tau_i), \tag{4}$$

where, for each $i = 1, \ldots, n+1$, $C^i := C^i(R_L) : \mathbb{R} \to \mathbb{R}^{1\times m}$ and $D^i := D^i(R_L) : \mathbb{R} \to \mathbb{R}^{1\times r}$ are given matrix-valued functions of the load resistance. As before, each of these functions is assumed to be continuously differentiable. Also, at the terminal time, we set $y(T) := y(T^-)$.

If the switching instants are chosen *a priori* — that is, the components of a given $\boldsymbol{\tau} \in \Gamma$ are used for the topology switching instants — then the state voltage of the power converter will evolve according to the switched dynamical system (1)–(3). Let $\mathbf{x}(\cdot|\boldsymbol{\tau}) := \mathbf{x}(\cdot|\boldsymbol{\tau}, \boldsymbol{\sigma}, R_L)$ denote this state voltage. The corresponding output voltage from (4) is denoted by $y(\cdot|\boldsymbol{\tau}) := y(\cdot|\boldsymbol{\tau}, \boldsymbol{\sigma}, R_L)$. Clearly, different choices of switching instants will result in different output voltage profiles.

Recall that the DC/DC power converter should (ideally) deliver a steady voltage to the attached appliance. Hence, the switching instants should be chosen so that the resulting output voltage ripple,

$$\sup_{t\in[0,T]} y(t|\boldsymbol{\tau}) - \inf_{t\in[0,T]} y(t|\boldsymbol{\tau}),$$