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Identification with stochastic sampling time jitter☆

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Abstract

This work investigates how stochastic sampling jitter noise affects the result of system identification, and proposes a modification of known approaches to mitigate the effects of sampling jitter, when the jitter is unknown and not directly measurable. By just assuming conventional additive measurement noise, the analysis shows that the identified model will get a bias in the transfer function amplitude that increases for higher frequencies. A frequency domain approach with a continuous-time model allows an analysis framework for sampling jitter noise. The bias and covariance in the frequency domain model are derived. These are used in bias compensated (weighted) least squares algorithms, and by asymptotic arguments this leads to a maximum likelihood algorithm. Continuous-time output error models are used for numerical illustrations.

Keywords: Non-uniform sampling; Sampling jitter; System identification; Stochastic systems; Maximum likelihood; Least squares estimation; Frequency domain; Parametric model

1. Introduction

Consider a deterministic signal model $s(t;\theta)$, which may depend on an observed or known input. This work studies the problem of identifying the unknown parameter vector θ, when the discrete time observations y_k requested at time $t = kT$ (T denotes the sampling interval) are subject both to the usual additive measurement noise and also stochastic unmeasurable jitter noise τ_k as part of the sampling process. That is, the observation includes the term $s(kT + \tau_k;\theta)$, which becomes a stochastic variable.

This type of non-uniform sampling may occur when uniform sampling is requested, but the sensor for one or several reasons cannot measure exactly at that time instant, and the true sampling time is either unmeasurable, or the communication protocol does not allow to transport time stamps to each measurement. Sampling jitter may also occur due to imperfect hold circuits, synchronization or other hardware problems. Not even high-performance digital oscilloscopes are free from sampling jitter as demonstrated in Verbeyst, Rolain, Schoukens, and Pintelon (2006). There, a dedicated system identification experiment is developed to estimate jitter effects. The result, when a sampling time of 1.22 ps is used, is that a commercial sampling oscilloscope has a sampling jitter standard deviation of around 1 ps, that is ≈ 80% of the sampling time.

The general problem of non-uniform sampling is extensively treated in literature, see Bilinskis and Mikelsons (1992) and Marvasti (2001). In most publications, the sampling times are known, and the problem is to analyze leakage and alias effects. Another twist is to design sampling times to minimize aliasing. For stochastic sampling jitter, the distribution of $s(t+\tau_k)$ is derived in Eng and Gustafsson (2005, 2006) and Souders, Flach, Hagwood, and Yang (1990). These results will be used and extended in this paper. In the context of jitter estimation, the sampling oscilloscope has been extensively studied, the methods include averaging over several measurements, as in Verspecht (1995), and Taylor series expansions, in Verbeyst et al. (2006), in order to estimate the variance of the jitter and use this to

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compensate for the jitter effects. In this paper, we consider a model-based approach, where the main goal is to find an underlying continuous-time system, and the jitter density can be seen as known or given in the estimation as a by-product.

The basic idea is as follows. The frequency domain approach is to minimize the distance between the measurement and signal model discrete time Fourier transforms (DTFT), $\|\mathrm{DTFT}(y_k)-\mathrm{DTFT}(s(kT+\tau_k;\theta))\|$ with respect to the parameters θ in some suitable norm. This frequency domain approach is standard in system identification (Ljung, 1999; Pintelon & Schoukens, 2001). A continuous-time model for $s(t)$ is used to be able to analyze the sampling jitter effects. The analysis shows that by neglecting the jitter, the Fourier transform (FT) of the signal model will suffer from an amplitude bias in $|\mathrm{FT}(s(t))| = |S(f)|$ that increases for higher frequencies. Further, the larger jitter noise variance, the larger bias. The remedy is to compensate for the bias, and the closed form solution involves a frequency weighting in the norm, $\|\mathrm{DTFT}(y_k) - \int S(\psi;\theta) w(f,\psi)\,\mathrm{d}\psi\|$.

The outline is as follows. The system identification problem and main notation are presented in Section 2. In Section 3, the bias effect of sampling jitter noise on the frequency transform is derived, and the bias compensated least squares (LS) estimator is proposed. Section 4 derives the second order properties of the frequency transform due to jitter noise, and a weighted LS algorithm as well as an asymptotic maximum likelihood (ML) estimator are presented. Section 5 illustrates these algorithms for several simulated numerical examples. The work is concluded in Section 6.

2. Problem formulation

The general problem formulation looks as follows. The sensor is requested to sample uniformly, but delivers discrete time measurements corrupted by amplitude noise and sampling time jitter according to

$$y_k = s(kT+\tau_k;\theta) + v(kT+\tau_k;\theta). \tag{1a}$$

The signal term, noise term, and jitter distribution can all be dependent on the unknown parameter θ and given by

$$s(t;\theta) = (g_\theta \star u)(t), \tag{1b}$$

$$v(t;\theta) = (h_\theta \star e)(t), \tag{1c}$$

$$\tau_k \in p_\theta(\tau). \tag{1d}$$

Here $u(t)$ is a known input, $e(t)$ is white noise with known characteristics, $g_\theta(t)$ denotes the system impulse response and $h_\theta(t)$ the noise dynamics. The jitter sampling noise is a sequence of independent stochastic variables with probability density function (pdf) $p_\theta(\tau)$. Both the signal, noise and sampling models can be parameterized in the unknown parameter vector θ. We will primarily focus on continuous-time systems here.

The system identification problems under consideration can be stated as estimating the parameter θ in a model structure

$$\mathscr{M}_{\mathrm{OE}}: g_\theta(t),\, h_\theta(t)=\delta(t),\, p_\theta(\tau)=p(\tau), \tag{2a}$$

$$\mathscr{M}_{\mathrm{BJ}}: g_\theta(t),\, h_\theta(t),\, p_\theta(\tau)=p(\tau), \tag{2b}$$

$$\mathscr{M}_{\mathrm{JOE}}: g_\theta(t),\, h_\theta(t)=\delta(t),\, p_\theta(\tau), \tag{2c}$$

$$\mathscr{M}_{\mathrm{JBJ}}: g_\theta(t),\, h_\theta(t),\, p_\theta(\tau). \tag{2d}$$

Here, OE denotes the output error and BJ the Box–Jenkins model structure, respectively, where the jitter distribution is known. JOE and JBJ are the corresponding problems where also the jitter noise distribution is parameterized.

Using previous knowledge about the sampling jitter effect in the frequency domain indicates that the frequency domain approach (see for example Ljung, 1999; Pintelon & Schoukens, 2001) is suitable for identification in this case. Denote the FT of the measurements and signal model, respectively,

$$Y_{\mathrm{d}}(f) = \sum_{k=0}^{N-1} y_k \mathrm{e}^{-\mathrm{i}2\pi f k T}, \tag{3}$$

$$S(f;\theta) = G(f;\theta)U(f). \tag{4}$$

The general problem formulation is now to minimize the distance between the measurement, $Y_{\mathrm{d}}(f)$, and model, $S(f;\theta)$, in the frequency domain.

$$\hat{\theta} = \arg\min_\theta \int_{-\infty}^{\infty} \lambda(f;\theta)|Y_{\mathrm{d}}(f) - S(f;\theta)|^2\,\mathrm{d}f, \tag{5}$$

for some suitable weighting function, λ. Normally, the weights, $\lambda(f;\theta)$ are given by the inverse noise spectrum (Ljung, 1999; Pintelon & Schoukens, 2001). We will show a few other examples later.

Given a continuous-time signal model $S_{\mathrm{c}}(f)$, a well known property of the FT gives that the discrete FT, $S_{\mathrm{d}}(f)$, becomes

$$S_{\mathrm{d}}(f) = \int_{-\infty}^{\infty} S_{\mathrm{c}}(\psi) d_N(f-\psi)\,\mathrm{d}\psi. \tag{6}$$

Here $d_N(f)$ is the normalized Dirichlet kernel (also known as the aliased sinc function), defined as

$$d_N(f) = \mathrm{e}^{-\mathrm{i}\pi f(N-1)T}\frac{\sin(\pi f N T)}{\sin(\pi f T)}. \tag{7}$$

The local behavior of the normalized Dirichlet kernel (see Fig. 1) describes the effects of leakage and its $1/T$ periodicity describes aliasing. For the regular case, with no sampling jitter, the correct way is to compare $Y_{\mathrm{d}}(f)$ with $S_{\mathrm{d}}(f;\theta)$, and using the unweighted LS norm in (5) over a discrete set of frequencies yields the parameter vector θ as

$$\hat{\theta}^{\mathrm{LS}} = \arg\min_\theta \sum_f \left| Y_{\mathrm{d}}(f) - \int_{-\infty}^{\infty} S_{\mathrm{c}}(\psi;\theta) d_N(f-\psi)\,\mathrm{d}\psi \right|^2. \tag{8}$$