Available online at www.sciencedirect.com

ScienceDirect

automatica

Automatica 44 (2008) 799–806

www.elsevier.com/locate/automatica

Brief paper

Robust H_∞ observer design for sampled-data Lipschitz nonlinear systems with exact and Euler approximate models[☆]

Masoud Abbaszadeh, Horacio J. Marquez*

Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4

Received 16 May 2006; received in revised form 9 June 2007; accepted 17 July 2007
Available online 11 December 2007

Abstract

An LMI approach is proposed for the design of robust H_∞ observers for a class of Lipschitz nonlinear systems. Two type of systems are considered, Lipschitz nonlinear discrete-time systems and Lipschitz nonlinear sampled-data systems with Euler approximate discrete-time models. Observer convergence when the exact discrete-time model of the system is available is shown. Then, practical convergence of the proposed observer is proved using the Euler approximate discrete-time model. As an additional feature, maximizing the admissible Lipschitz constant, the solution of the proposed LMI optimization problem guaranties robustness against some nonlinear uncertainties. The robust H_∞ observer synthesis problem is solved for both cases. The maximum disturbance attenuation level is achieved through LMI optimization.

Keywords: Lipschitz nonlinear systems; Robust observers; H_∞ filtering; Euler discretization; LMI optimization

1. Introduction

Design of discrete-time nonlinear observers has been the subject of significant attention in recent years. See for example Califano, Monaco, and Normand-Cyrot (2003), Xiao, Kazantzis, Kravaris, and Krener (2003), Kazantzis and Kravaris (2001) and Wang and Unbehauen (2000) as well as the sampled data nonlinear observers of Moraal and Grizzle (1995), Biyik and Arcak (2006) and Laila and Astolfi (2006). The study of the nonlinear discrete-time observers is important at least for two reasons. First, most continuous-time control system designs are implemented digitally. Given that in most practical cases it is impossible to measure every state variable in real time, these controllers require the reconstruction of the states of the discrete-time model of the true continuous-time plant. Second, there are systems which are inherently discrete-time and do not originate from discretization of a continuous-time plant. Of those, discrete-time observers of continuous-time systems are particularly challenging. The reason is that exact discretization of a continuous-time nonlinear model is usually not possible to obtain. Approximate discrete-time models, on the other hand, are affected by the consequent approximation error. In this paper, we address both problems. First, we consider a class of nonlinear discrete-time systems with exact model. A nonlinear H_∞ observer design algorithm is proposed for these systems based on an LMI approach. Then, the nonlinear sampled data system, sampled using a zero-order hold device, with Euler approximate model is considered. The Euler approximation is important because not only it is easy to derive but also it maintains the structure of the original nonlinear model. We show that by appropriate selection of one of the parameters in our proposed LMIs (the only design parameter in our algorithm), the practical convergence of the observer via Euler approximation is guaranteed as well as the robust H_∞ cost. Our approach is based on the recent results of Arcak and Nešić (2004). See Assoudi, Yaagoubi, and Hammouri (2002) and Busawon, Saif, and Leon-Morales (1999) for other approaches. We emphasize that while the algorithms in Assoudi et al. (2002) and Busawon et al. (1999) are specifically designed

☆ This paper was not presented at any IFAC meeting. An early version of this work will be presented at the 2007 ACC. This paper was recommended for publication in revised form by Associate Editor Murat Arcak under the direction of Editor Hassan Khalil. This research was supported by the Natural Sciences and Engineering Council of Canada (NSERC).

* Corresponding author. Tel.: +1 780 492 3333; fax: +1 780 492 8506.
E-mail addresses: masoud@ece.ualberta.ca (M. Abbaszadeh), marquez@ualberta.ca (H.J. Marquez).

0005-1098/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.automatica.2007.07.021

for Euler discretization, our proposed algorithm can be applied either to the nominal exact discrete-time model or to its Euler approximation.

The LMI based observer design for uncertain discrete-time systems has been addressed in several works e.g. Xu (2002) and Lu and Ho (2004). In all these studies, the proposed LMIs are nonlinear in the Lipschitz constant and thus it cannot be considered as one of the LMI variables. In the algorithm proposed here, first the problem is addressed in the general case, then, having a bound on the Lipschitz constant, the LMIs become linear in the Lipschitz constant and we can take advantage of this feature to solve an optimization problem over it. Provided that the optimal solution is larger than the actual Lipschitz constant of the system in hand, we show that the redundancy achieved can guarantee robustness against some nonlinear uncertainty in the original continuous-time model for both exact and Euler approximate discretizations. Therefore, unlike the aforementioned LMI approaches in which the uncertainty is in the linear part of the model, here the uncertainty can be in the nonlinear part as well as the whole model due to approximate discretization. The paper is organized as follows: In Section 2, an observer design method for a class of nonlinear discrete-time systems is introduced. In Section 3 the practical convergence of the proposed observer via the Euler approximate models is shown. In Section 4, the results of the two previous sections will extend into the H_∞ context followed by an illustrative example showing satisfactory performance of our algorithm.

2. Observer design for nonlinear discrete-time systems

We consider the following system:

$$x_{k+1} = A_d x_k + F(x_k, u_k), \tag{1}$$

$$y_k = C_d x_k, \tag{2}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p$ and $F(x_k, u_k)$ contains nonlinearities of second order or higher. The above system can be either an inherently discrete-time system or the exact discretization of a continuous-time system. We assume that $F(x_k, u_k)$ is locally Lipschitz with respect to x in a region $\mathscr{D}$, uniformly in u, i.e. $\forall\, x_{1k}, x_{2k} \in \mathscr{D}$:

$$\|F(x_1, u^*) - F(x_2, u^*)\| \leqslant \gamma_d \|x_1 - x_2\|, \tag{3}$$

where $\|.\|$ is the induced 2-norm, u^* is any admissible control sequence and $\gamma_d > 0$ is called the Lipschitz constant. If the nonlinear function F satisfies the Lipschitz continuity condition globally in $\mathbb{R}^n$, then all the results in this and the ensuing sections will be valid globally. The proposed observer is in the following form:

$$\hat{x}_{k+1} = A_d \hat{x}_k + F(\hat{x}, u) + L(y_k - C_d \hat{x}_k). \tag{4}$$

Defining the observer error as $e_k \triangleq x_k - \hat{x}_k$, we have

$$e_{k+1} = (A_d - LC_d)e_k + F(x_k, u_k) - F(\hat{x}_k, u_k). \tag{5}$$

Our goal in this section is two-fold: (i) In the first place, we want to find an observer gain, L, such that the observer error dynamics is asymptotically stable. (ii) We want to maximize γ_d, the allowable Lipschitz constant of the nonlinear system. The following theorem addresses the first goal.

Theorem 1. *Consider the system* (1)–(2) *with given Lipschitz constant* γ_d. *The observer error dynamics* (5) *is* (*globally*) *asymptotically stable if there exist scalar* $\varepsilon > 0$, *fixed matrix* $Q > 0$ *and matrices* $P > 0$ *and* G *such that the following set of LMIs has a solution*:

$$\Xi \triangleq \begin{bmatrix} P - Q - \varepsilon I & A_d^{\mathrm{T}} P - C_d^{\mathrm{T}} G^{\mathrm{T}} \\ PA_d - GC_d & P \end{bmatrix} > 0, \tag{6}$$

$$\begin{bmatrix} \Psi_1 I & P \\ P & \Psi_1 I \end{bmatrix} > 0, \tag{7}$$

where

$$\Psi_1 = \frac{-\lambda_{\max}(Q) + \sqrt{\lambda_{\max}^2(Q) + \dfrac{1}{\gamma_d^2}\lambda_{\min}^2(Q)}}{\gamma_d + 2}. \tag{8}$$

P, G, *and* ε *are the LMI variables and* Q *is a design parameter to be chosen. Once the problem is solved*: $L = P^{-1}G$.

Proof. Consider the Lyapunov function candidate as $V_k = e_k^{\mathrm{T}} P e_k$, then

$$\begin{aligned} \Delta V = V_{k+1} - V_k &= e_k^{\mathrm{T}}(A_d - LC_d)^{\mathrm{T}} P (A_d - LC_d) e_k \\ &\quad + 2e_k^{\mathrm{T}}(A_d - LC_d)^{\mathrm{T}} P (F_k - \hat{F}_k) \\ &\quad + (F_k - \hat{F}_k)^{\mathrm{T}} P (F_k - \hat{F}_k) - e_k^{\mathrm{T}} P e_k, \end{aligned} \tag{9}$$

where we use notations $F_k \triangleq F(x_k, u_k)$, $\hat{F}_k \triangleq F(\hat{x}_k, u_k)$. Suppose $\exists P$, $Q > 0$ such that the following discrete-time Lyapunov equation has a solution:

$$(A_d - LC_d)^{\mathrm{T}} P (A_d - LC_d) - P = -Q. \tag{10}$$

Then (9) becomes

$$\begin{aligned} \Delta V = &- e_k^{\mathrm{T}} Q e_k + 2e_k^{\mathrm{T}}(A_d - LC_d)^{\mathrm{T}} P (F_k - \hat{F}_k) \\ &+ (F_k - \hat{F}_k)^{\mathrm{T}} P (F_k - \hat{F}_k). \end{aligned} \tag{11}$$

Using Rayleigh and Schwarz inequalities, we have

$$\|e_k^{\mathrm{T}} Q e_k\| \geqslant \lambda_{\min}(Q)\|e_k\|^2, \tag{12}$$

$$\begin{aligned} &\|2e_k^{\mathrm{T}}(A_d - LC_d)^{\mathrm{T}} P (F_k - \hat{F}_k)\| \\ &\quad \leqslant \|2e_k^{\mathrm{T}} P (F_k - \hat{F}_k)\| \cdots \|A_d - LC_d\| \\ &\quad \leqslant 2\gamma_d \lambda_{\max}(P)\|e_k\|^2 \|A_d - LC_d\| \\ &\quad = 2\gamma_d \lambda_{\max}(P)\|e_k\|^2 \bar{\sigma}(A_d - LC_d), \end{aligned} \tag{13}$$

$$\begin{aligned} \|(F_k - \hat{F}_k)^{\mathrm{T}} P (F_k - \hat{F}_k)\| &\leqslant \lambda_{\max}(P)\|(F_k - \hat{F}_k)\|^2 \\ &\leqslant \gamma_d^2 \lambda_{\max}(P)\|e_k\|^2. \end{aligned} \tag{14}$$

So for $\Delta V < 0$ it is sufficient to have

$$-\lambda_{\min}(Q) + \lambda_{\max}(P)[2\gamma_d \bar{\sigma}(A_d - LC_d) + \gamma_d^2] < 0. \tag{15}$$

Condition (15) along with (10) are sufficient conditions for asymptotic stability. We now endeavor to convert these