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Brief paper

Global stabilization of a class of partially known nonnegative systems[☆]

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Abstract

In this paper we deal with the problem of global output feedback stabilization of a class of n-dimensional nonlinear nonnegative systems possessing a one-dimensional analytically unknown part that is also a measured output. We first propose our main result, an output feedback control procedure, taking advantage of measurements of the uncertain part, able to globally stabilize the system toward an adjustable equilibrium point in the positive orthant. Though quite general, this result is based on hypotheses that might be difficult to check in practice. Then in a second step, through a theorem on a class of nonnegative systems linking the existence of a positive equilibrium to its global asymptotic stability, we propose other hypotheses for our main result to hold. These new hypotheses are more restrictive but much simpler to check. An illustrative example highlights both the potentially complex open loop dynamics of the considered systems and the interesting characteristics of the control procedure.

Keywords: Positive systems; Partially known systems; Global stabilization

1. Introduction

Nonnegative systems of ordinary differential equations are systems generating trajectories, that, if initiated in the closure of the positive orthant of $\mathbb{R}^n$, remain in this set for all positive times. Such systems are being paid much attention since they abound in many applied areas such as life sciences, social sciences, chemical sciences, telecommunications, traffic flows etc. (Benvenuti, De Santis, & Farina, 2003; Commault & Marchand, 2006; Farina & Rinaldi, 2000; Luenberger, 1979).

In this paper we deal with the problem of global output feedback stabilization of a class of n-dimensional nonlinear nonnegative systems possessing a one-dimensional analytically unknown part that is also a measured output. Our main result generalizes an earlier one obtained in lower dimensions and for simpler structures in the context of bioprocess control (Mailleret, Bernard, & Steyer, 2004).

This paper is organized as follows. We first introduce some notations and definitions related to positivity for vectors and dynamical systems. We then introduce the class of uncertain nonnegative systems that are concerned with our control problem. We propose an output feedback procedure, taking advantage of measurements of the uncertain part, which is able to globally stabilize the system toward an adjustable equilibrium point in the interior of the positive orthant. The hypotheses on which our main result is built might however be difficult to verify in practice. In order to derive some hypotheses that are much simpler to check, we propose a theorem on a class of positive systems that links the existence of a positive equilibrium to its global asymptotic stability (GAS). An illustrative example concludes the article, showing the potentially complex open loop dynamics that might be produced by the systems under study as well as the control efficiency.

2. Preliminaries

We consider the following autonomous nonlinear dynamical systems in $\mathbb{R}^n$:

$$\dot{x} = f(x). \tag{1}$$

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For the sake of simplicity, we assume that the functions encountered throughout the paper are sufficiently smooth.

In what follows, we will use the following notations:

- $x(t, x_0)$ denotes the forward orbit at time t of system (1) initiated at x_0.
- $Df(x)$ denotes the Jacobian matrix of system (1) at state x.
- $\mathcal{L}_f$ denotes the Lie derivative operator along the vector field defined by system (1).
- $s(A)$ denotes the stability modulus of a matrix A, *i.e.* A's dominant eigenvalue real part.
- $x_{|x_i=0}$ denotes a state vector whose ith component equals zero.

Let us first state what we refer to as nonnegativity (resp. positivity) for vectors and dynamical systems in $\mathbb{R}^n$.

Definition 1. $x \in \mathbb{R}^n$ is nonnegative (resp. positive) and denoted $x \geq 0$ (resp. $x \gg 0$) iff all its components are nonnegative (resp. positive).

Same definitions of the relations "$\geq$" and "$\gg$" (i.e. component by component) will be used for matrices too. We denote $\mathbb{R}^n_+$ and $\mathrm{int}(\mathbb{R}^n_+)$ the sets of nonnegative and positive vectors of $\mathbb{R}^n$, respectively.

We now state the definition of nonnegative systems as well as the definition of a special class of such systems, the Boundary Repelling Positive (BRP) ones.

Definition 2. System (1) is a nonnegative (resp. BRP) system iff:

$$\forall i \in [1 \dots n], \quad \forall x_{|x_i=0} \geq 0, \quad \dot{x}_i(x_{|x_i=0}) = f_i(x_{|x_i=0}) \geq 0, \quad (\text{resp. } f_i(x_{|x_i=0}) > 0).$$

It is straightforward that Definition 2 implies:

Proposition 1. *Consider a nonnegative (resp.* BRP*) system* (1). *Then:*

$$\forall (x_0, t) \in \mathbb{R}^n_+ \times \mathbb{R}^+ \quad (\textit{resp.}\ \forall (x_0, t) \in \mathbb{R}^n_+ \times (0, +\infty)), \quad x(t, x_0) \geq 0 \quad (\textit{resp.}\ x(t, x_0) \gg 0).$$

Remark 1. BRP systems are a special case of "strongly positive systems" introduced by Luenberger (1979) (these systems produce trajectories which, initiated in the nonnegative orthant, remain in the positive orthant for all positive time).

3. Main result

In the following we focus on the (global) asymptotic stabilization problem of the following class of nonlinear nonnegative uncertain systems in $\mathbb{R}^n$:

$$\begin{aligned} \dot{x} &= u f(x) + \psi(x) c, \\ y &= \psi(x). \end{aligned} \tag{2}$$

- $u \geq 0$ being the scalar input of system (2).
- $f : \mathbb{R}^n \to \mathbb{R}^n$
- $c \in \mathbb{R}^n$
- $\psi : \mathbb{R}^n \to \mathbb{R}$ is an output of system (2)

Function $\psi(.)$ features the *uncertain/unknown* part of the system. For control purposes, we suppose that, though analytically unknown, $\psi(.)$ might be measured online. We moreover assume the following:

Hypotheses 1 (*H1*).

(1) $\forall x \in \mathrm{int}(\mathbb{R}^n_+)$, $\psi(x) > 0$ and $\forall i$, $\psi(x_{|x_i=0}) c_i \geq 0$
(2) $f(.)$ is such that the system $\dot{x} = f(x)$ is nonnegative
(3) $\exists \beta_m \in \mathbb{R}^+$ such that $\forall \beta > \beta_m$, the system:

$$\dot{x} = \beta f(x) + c. \tag{3}$$

is a BRP system and possesses an equilibrium $x^\star_\beta \gg 0$ which is GAS on $\mathbb{R}^n_+$.

These hypotheses will be commented after the statement of our main result. Just notice that Hypotheses H1-1 and H1-2 together with $u \geq 0$ ensure the nonnegativity of the considered class of systems (2).

We are now ready to state our main result:

Theorem 1. *Consider a system* (2) *and let Hypotheses H1 hold. Then for all* $\gamma > \beta_m$, *the nonlinear control law:*

$$u = \gamma y = \gamma \psi(x). \tag{4}$$

globally stabilizes system (2) *on* $\mathrm{int}(\mathbb{R}^n_+)$ *toward the positive equilibrium* $x^\star_\gamma$ *such that:*

$$f(x^\star_\gamma) = \frac{-1}{\gamma} c.$$

To prove Theorem 1, we need the following lemma:

Lemma 1. *A nonnegative system* (1) *possesses a positive equilibrium point* $x^\star$ GAS *on* $\mathrm{int}(\mathbb{R}^n_+)$ *iff there exists a smooth real valued function* $V(x)$ *such that:*

$$\begin{aligned} &\forall x \gg 0, \quad x \neq x^\star, \quad V(x) > 0, \\ &V(x^\star) = 0, \\ &\forall x \gg 0, \quad x \neq x^\star, \quad \mathcal{L}_f V(x) < 0, \\ &\lim_{\|\ln(x)\| \to +\infty} V(x) = +\infty. \end{aligned}$$

Proof (Lemma 1). Consider a nonnegative system (1) and suppose it possesses a positive equilibrium $x^\star$ GAS on $\mathrm{int}(\mathbb{R}^n_+)$. Now consider the following increasing (in the "$\gg$" sense) change of variables:

$$z = \ln\left(\frac{x}{x^\star}\right) = \left(\ln\left(\frac{x_1}{x^\star_1}\right), \dots, \ln\left(\frac{x_n}{x^\star_n}\right)\right)^T \tag{5}$$

that maps $\mathrm{int}(\mathbb{R}^n_+)$ on $\mathbb{R}^n$ and $x^\star$ to 0. Then we have:

$$\dot{z} = \frac{f(x^\star e^z)}{x^\star} e^{-z}. \tag{6}$$

Since $x^\star$ is a GAS equilibrium on $\mathrm{int}(\mathbb{R}^n_+)$ for (1), so is 0 on $\mathbb{R}^n$ for system (6). Then, from Kurzweil's converse Lyapunov theorem on GAS (Kurzweil, 1956), there exists a smooth