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Robust state prediction for descriptor systems[☆]

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Abstract

This paper deals with the problem of state prediction for descriptor systems subject to bounded uncertainties. The problem is stated in terms of the optimization of an appropriate quadratic functional. This functional is well suited to derive not only the robust predictor for descriptor systems but also that for usual state-space systems. Numerical examples are included in order to demonstrate the performance of this new filter.

Keywords: Descriptor systems; Filtering; Robust estimation; Kalman filter

1. Introduction

For usual state-space systems, generalizations of the classical Kalman filter to encompass systems with bounded uncertainties have been the focus of a number of papers (see, for instance, Sayed (2001) and Shaked and de Souza (1995), and references therein). For descriptor systems without uncertainties, there has been an intensive study on the Kalman filtering problem (see Darouach, Zasadzinski, and Mehdi (1993), Deng and Liu (1999), Keller, Nowakowski, and Darouach (1992), Nikoukhah, Campbell, and Delebecque (1999), Nikoukhah, Willsky, and Levy (1992), Xi (1997), Zhang, Chai, and Liu (1998) and Zhang, Xie, and Soh (1999, 2003)). Different formulations have been proposed in order to deal with this problem. In a direct descriptor context, one can consider the least squares method (Darouach et al., 1993; Keller et al., 1992), the maximum likelihood criterion (Nikoukhah et al., 1999, 1992), the minimum-variance estimation (Germani, Manes, & Palumbo, 2001), and the ARMA innovation model (Deng and Liu, 1999; Zhang, Xie, & Soh, 1998; Zhang et al., 1999).

This rich volume of work is, in part, justified by the fact that not all properties and results for usual state-space systems remain valid for descriptor systems. For instance, it is well known that, for usual state-space systems without uncertainties, the predicted estimates recursion and filtered estimates recursion are just different formulations of the same filter (Kailath, Sayed, & Hassibi, 2000). However, this is not true for descriptor systems: in general, there is no rearrangement that transforms the predictor recursion into the filter recursion, or vice versa (Ishihara, Terra, & Campos, 2005).

The subject of this paper is the robust prediction of the one step ahead descriptor variable of an uncertain discrete-time linear stochastic descriptor system

$$\begin{aligned}(E_{i+1}+\delta E_{i+1})x_{i+1} &= (F_i+\delta F_i)x_i+w_i,\\ &\quad i=0,1,\ldots,N\\ z_i &= (H_i+\delta H_i)x_i+v_i \qquad (1)\end{aligned}$$

where $x_i \in \Re^n$ is the descriptor variable, $z_i \in \Re^p$ is the measured output, $w_i \in \Re^m$ and $v_i \in \Re^p$ are the process and measurement noises, $E_{i+1} \in \Re^{m\times n}$, $F_i \in \Re^{m\times n}$ and $H_i \in \Re^{p\times n}$ are the known nominal system matrices, and δE_{i+1}, δF_i and δH_i are time-varying perturbations of the nominal system matrices. More precisely, given a sequence of measured outputs $\{z_0, z_1, \ldots, z_i\}$, the main objective is to develop the best estimate for the descriptor variable x_{i+1} taking into account the worst-case uncertainty. The uncertainties in the system matrices E_{i+1}, F_i, and H_i of the model (1) are assumed to have the following structure:

$$\delta F_i = M_{f,i}\Delta_i N_{f,i}; \qquad \delta E_{i+1} = M_{f,i}\Delta_i N_{e,i+1};$$

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$$\delta H_i = M_{h,i} \Delta_i N_{h,i}; \quad \|\Delta_i\| \leq 1 \tag{2}$$

where $M_{f,i}$, $M_{h,i}$, $N_{e,i+1}$, $N_{f,i}$, $N_{h,i}$ are known matrices and Δ_i is an unknown bounded matrix (with the norm less than or equal to 1).

The approach used in this paper to deduce the robust descriptor predictor filter is based on the arguments developed in Sayed (2001). The filter developed here is based on a new quadratic functional suited for the predicted form. When the new filter is specialized for the usual state-space systems, it provides an alternative robust predictor to the one proposed in Sayed (2001) (see more details in Remark 3.3).

This paper is organized as follows. In Section 2, the descriptor Kalman predictor filter for systems without uncertainties is revisited. In Section 3, a solution for the recursive robust fitting problem as a generalization of the Kalman filter for uncertain descriptor systems is proposed. In Section 4, simulation results are presented to demonstrate the performance of the descriptor robust filter developed in this paper.

2. Optimum data fitting and the standard predictor filter for descriptor systems

In this section, in order to obtain some insight in the solution of the robust prediction problem, we revise the predictor problem for the descriptor system without uncertainties. Consider the system

$$E_{i+1}x_{i+1} = F_i x_i + w_i, \quad i = 0, 1, \ldots, N$$
$$z_i = H_i x_i + v_i. \tag{3}$$

The initial condition and the process and measurement noises, $\{x_0, w_i, v_i\}$, are assumed to be uncorrelated zero-mean random variables with second-order statistics

$$\operatorname{cov}\left(\begin{bmatrix} x_0^T & w_i^T & v_i^T \end{bmatrix}^T\right) = \operatorname{diag}(P_0, Q_i\delta_{ij}, R_i\delta_{ij}) > 0, \tag{4}$$

where $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ otherwise. The solution of the one step descriptor predictor problem is usually stated as a recursive algorithm. For easy reference, the predictor algorithm is presented in the following theorem.

Theorem 2.1 (*Ishihara, Terra, & Campos, 2004; Nikoukhah et al., 1999*). *Suppose that E_i has full column rank for all $i \geq 0$. The optimum predicted estimates $\widehat{x}_{i+1|i}$ can be obtained recursively by updating from $\{\hat{x}_{i|i-1}, P_{i|i-1}\}$ to $\{\hat{x}_{i+1|i}, P_{i+1|i}\}$ as follows*

$$P_{i+1|i} := \left(\begin{bmatrix} E_{i+1} \\ 0 \end{bmatrix}^T \begin{bmatrix} Q_i + F_i P_{i|i-1} F_i^T & -F_i P_{i|i-1} H_i^T \\ -H_i P_{i|i-1} F_i^T & R_i + H_i P_{i|i-1} H_i^T \end{bmatrix}^{-1} \times \begin{bmatrix} E_{i+1} \\ 0 \end{bmatrix}\right)^{-1}$$

$$\hat{x}_{i+1|i} := P_{i+1|i} \begin{bmatrix} E_{i+1} \\ 0 \end{bmatrix}^T \times \begin{bmatrix} Q_i + F_i P_{i|i-1} F_i^T & -F_i P_{i|i-1} H_i^T \\ -H_i P_{i|i-1} F_i^T & R_i + H_i P_{i|i-1} H_i^T \end{bmatrix}^{-1} \times \begin{bmatrix} F_i \hat{x}_{i|i-1} \\ z_i - H_i \widehat{x}_{i|i-1} \end{bmatrix}$$

with the initial conditions $P_{0|-1} := P_0$ and $\hat{x}_{0|-1} := \bar{x}_0 = 0$. $\diamond$

For $E_{i+1} = I$, the recursions of Theorem 2.1 reduce to the well known Kalman recursions for the one step predictor estimates (cf. e.g. Kailath et al. (2000)). Although the descriptor predictor has been considered only by few works (Ishihara et al., 2004; Nikoukhah et al., 1999; Zhang et al., 1999), in principle, similarly to the descriptor filter, it can be derived by many different approaches such as maximum likelihood, minimum variance, ARMA innovation model or least square (see Darouach et al. (1993), Deng and Liu (1999), Keller et al. (1992), Nikoukhah et al. (1999, 1992), Xi (1997), Zhang, Chai et al. (1998) and Zhang et al. (1999, 2003)).

In Ishihara et al. (2004), it is shown that the standard descriptor Kalman predictor filter of Theorem 2.1 can be obtained with data fitting arguments. In this formulation, a deterministic optimization problem which corresponds to the original stochastic formulation is solved. This approach is convenient to provide not only the predicted estimate recursions, but also the filtered and smoothed estimate recursions. In Ishihara et al. (2005), it is shown that in order to update the optimum predicted estimate of x_i from $\hat{x}_{i|i-1}$ to $\hat{x}_{i+1|i}$, one can solve the following optimization problem

$$\min_{x_i, x_{i+1}} \left[\|x_i - \hat{x}_{i|i-1}\|^2_{P^{-1}_{i|i-1}} + \|E_{i+1}x_{i+1} - F_i x_i\|^2_{Q_i^{-1}} + \|z_i - H_i x_i\|^2_{R_i^{-1}} \right]. \tag{5}$$

Note that the functional in (5) is different from those previously considered for descriptor filtering (cf. the functionals of Darouach et al. (1993) and Keller et al. (1992)). All the minimizations of these different quadratic functionals lead to the nominal descriptor filter. However, (5) is the most appropriate in order to be generalized to the robust case as will be shown in the next section. The intuitive interpretation of (5) can be found in Ishihara et al. (2005).

3. Robust predictor for discrete time descriptor systems

The optimum robust fitting problem for the predicted estimate is defined as follows. It is assumed that at step $i-1$ one has *a priori* prediction for the next state x_i, denoted by $\hat{x}_{i|i-1}$, and there exists a positive-definite weighting matrix $P_{i|i-1}$ for the state estimation error $x_i - \hat{x}_{i|i-1}$. At the time i, in possession of the new observation z_i, the following robust data fitting problem is proposed in order to update the predicted estimate from $\hat{x}_{i|i-1}$ to $\hat{x}_{i+1|i}$: Solve for $i > 0$,