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Stability analysis of neutral systems with distributed delays☆

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Abstract

The stability of neutral systems with distributed delays is investigated in this paper. A modified Lyapunov–Krasovskii functional is constructed to study this class of systems. The proposed stability criterion is discrete-, distributed- and neutral-delay-dependent. In addition, by this method one can study the case when the coefficient matrix of the neutral delay term is time-varying uncertain. The reduced conservatism is illustrated in a numerical example.

Keywords: Time delay; Stability; Neutral systems; Distributed delay; Linear matrix inequality (LMI)

1. Introduction

The stability of linear neutral systems has received considerable attention in the last two decades. The research on this topic can be classified into two types, namely a time-domain approach and a frequency-domain approach. For the current study of linear neutral systems by the time-domain approach, the Lyapunov–Krasovskii method (Gu, Kharitonov, & Chen, 2003) is widely used, and the stability criteria are usually proposed in terms of linear matrix inequalities (LMIs) (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). Reducing the conservatism of the LMI conditions motivates the present research.

Stability analysis of neutral systems with distributed delays is of both practical and theoretical importance. For some systems, delay phenomena may not be simply considered as delays in the velocity terms and/or discrete delays in the states. Therefore, it is desirable to extend the system model to include distributed delays. Practical applications, modeled by systems with distributed delays, can be found in Fiagbedzi and Pearson (1987), Hale and Lunel (1993) and Richard (2003). However, there are only a few results available to check the stability of this class of systems. Gu (2003) and Gu, Han, Luo, and Niculescu (2001) use a discretized Lyapunov functional method to check the stability of systems with distributed delays. However this method is complicated and is difficult to extend to the synthesis problems.

In the recent papers by Chen and Zheng (2007) and Han (2004), a descriptor system approach (see Fridman (2001) and Fridman and Shaked (2002)) has been used to investigate the stability of neutral systems with discrete and distributed delays. Han (2004) rewrites the discrete-delay term and employs a decomposition technique (Goubet-Bartholoméüs, Dambrine, & Richard, 1997). Different from Han (2004), Chen and Zheng (2007) rewrite both the discrete-delay and the distributed-delay terms and apply Moon's inequality (Moon, Park, Kwon, & Lee, 2001). To the best of our knowledge, the stability criterion in Chen and Zheng (2007) is the least conservative among the existing ones. The stability conditions in Chen and Zheng (2007) and Han (2004) are discrete- and distributed-delay-dependent but neutral-delay-independent.

In order to further improve the results, one will naturally think of employing free weighting matrices. Free weighting matrices, also called slack matrix variables, have been used in many recent papers to study time-delay systems (e.g., He, Wu, She, and Liu (2004a,b)). He et al. (2004a) and Wu, He, and She (2004) study the stability of neutral systems by introducing free weighting matrices, and the results therein can be extended to the systems with distributed delays. However, by the methods in He et al. (2004a) and Wu et al. (2004) one cannot handle

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the case when the coefficient matrix of the neutral-delay term is time-varying uncertain.

The objective of this paper is to further reduce the conservatism of the stability conditions for linear neutral systems with distributed delays. Toward this end, a modified Lyapunov–Krasovskii functional is constructed and free weighting matrices are employed. The resultant stability criterion is less conservative. The reduced conservatism is illustrated in a numerical example.

Notation. Throughout this paper, A^{T} and A^{-1} denote the transpose and the inverse of a matrix A, respectively. $A > 0$ ($A < 0$) means that A is positive definite (negative definite). $*$ represents the blocks that are readily inferred by symmetry and I denotes the unit matrix of appropriate dimensions. In this paper, if not explicit, matrices are assumed to have compatible dimensions.

2. Problem statement

Consider the following neutral system with discrete and distributed delays:

$$\begin{aligned}\dot{x}(t) - C(t)\dot{x}(t-\tau) &= A(t)x(t) + B(t)x(t-h) \\ &\quad + D(t)\int_{t-r}^{t} x(s)\mathrm{d}s \\ x(t) = \phi(t), \quad & t \in [-\max\{\tau, h, r\}, 0]\end{aligned} \tag{1}$$

where $x(t)$ is the state, $\tau > 0$, $h > 0$ and $r > 0$ are constant neutral, discrete and distributed delay, respectively, $\phi(t)$ is the initial condition, $A(t)$, $B(t)$, $C(t)$ and $D(t)$ are uncertain matrices. We assume that the uncertainties are norm-bounded and can be described as:

$$\begin{aligned}&A(t) = A + \Delta A(t)\\ &B(t) = B + \Delta B(t)\\ &C(t) = C + \Delta C(t)\\ &D(t) = D + \Delta D(t)\\ &\begin{bmatrix}\Delta A(t) & \Delta B(t) & \Delta C(t) & \Delta D(t)\end{bmatrix}\\ &\quad = LF(t)\begin{bmatrix}E_A & E_B & E_C & E_D\end{bmatrix}\end{aligned}$$

where $A, B, C, D, L, E_A, E_B, E_C$ and E_D are known constant matrices and $F(t)$ is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying $\|F(t)\| \leq 1$. Throughout this paper, we assume that the matrix $C(t)$ is Schur stable.

In this paper, we will study the robust stability of system (1).

3. Main results

First, consider the nominal system of (1), that is the system

$$\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + Bx(t-h) + D\int_{t-r}^{t} x(s)\mathrm{d}s. \tag{2}$$

In order to analyze the stability of system (2), the following integral inequality is required.

Lemma 1 (Gu et al., 2003). *For any positive symmetric constant matrix M and a scalar $\gamma > 0$, if there exists a vector function $\omega : [0, \gamma] \to \mathbb{R}^n$ such that the integrals $\int_o^\gamma \omega^{\mathrm{T}}(s)M\omega(s)\mathrm{d}s$ and $\int_o^\gamma \omega^{\mathrm{T}}(s)\mathrm{d}s$ are well defined, then the following inequality holds:*

$$\gamma\int_o^\gamma \omega^{\mathrm{T}}(s)M\omega(s)\mathrm{d}s \geq \left(\int_o^\gamma \omega^{\mathrm{T}}(s)\mathrm{d}s\right) M \left(\int_o^\gamma \omega(s)\mathrm{d}s\right).$$

For the stability of system (2), we have the following result.

Theorem 1. *For given scalars τ, h and r, the neutral system (2) is asymptotically stable, if there exist matrices $P_{11} = P_{11}^{\mathrm{T}}$, P_{12}, P_{13}, $P_{22} = P_{22}^{\mathrm{T}}$, P_{23}, $P_{33} = P_{33}^{\mathrm{T}}$, $Q = Q^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, $S = S^{\mathrm{T}}$, S_1, $T = T^{\mathrm{T}}$, $U = U^{\mathrm{T}} > 0$, $V = V^{\mathrm{T}} > 0$, $W = W^{\mathrm{T}} > 0$, M_i, N_i and O_i $(i = 1, 2, \ldots, 6)$ satisfying the following LMIs:*

$$\begin{bmatrix}\Omega & hN & rP' & \tau O\\ * & -hR & 0 & 0\\ * & * & -rV & 0\\ * & * & * & -\tau W\end{bmatrix} < 0 \tag{3}$$

$$P = \begin{bmatrix}P_{11} & P_{12} & P_{13}\\ * & P_{22} & P_{23}\\ * & * & P_{33}\end{bmatrix} > 0, \qquad \begin{bmatrix}S & S_1\\ * & T\end{bmatrix} > 0 \tag{4ab}$$

where

$$\Omega = \begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16}\\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26}\\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36}\\ * & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46}\\ * & * & * & * & \Omega_{55} & \Omega_{56}\\ * & * & * & * & * & \Omega_{66}\end{bmatrix}$$

$$\begin{aligned}\Omega_{11} &= Q + S + rU + M_1A + A^{\mathrm{T}}M_1^{\mathrm{T}}\\ &\quad + N_1 + N_1^{\mathrm{T}} + O_1 + O_1^{\mathrm{T}}\\ \Omega_{12} &= A^{\mathrm{T}}M_2^{\mathrm{T}} + N_2^{\mathrm{T}} - O_1 + O_2^{\mathrm{T}}\\ \Omega_{13} &= M_1D + A^{\mathrm{T}}M_3^{\mathrm{T}} + N_3^{\mathrm{T}} + O_3^{\mathrm{T}}\\ \Omega_{14} &= P_{11} + S_1 - M_1 + A^{\mathrm{T}}M_4^{\mathrm{T}} + N_4^{\mathrm{T}} + O_4^{\mathrm{T}}\\ \Omega_{15} &= P_{12} + M_1C + A^{\mathrm{T}}M_5^{\mathrm{T}} + N_5^{\mathrm{T}} + O_5^{\mathrm{T}}\\ \Omega_{16} &= M_1B + A^{\mathrm{T}}M_6^{\mathrm{T}} - N_1 + N_6^{\mathrm{T}} + O_6^{\mathrm{T}}\\ \Omega_{22} &= -S - O_2 - O_2^{\mathrm{T}}\\ \Omega_{23} &= M_2D - O_3^{\mathrm{T}}\\ \Omega_{24} &= P_{12}^{\mathrm{T}} - M_2 - O_4^{\mathrm{T}}\\ \Omega_{25} &= P_{22} - S_1 + M_2C - O_5^{\mathrm{T}}\\ \Omega_{26} &= M_2B - N_2 - O_6^{\mathrm{T}}\\ \Omega_{33} &= -\frac{1}{r}U + M_3D + D^{\mathrm{T}}M_3^{\mathrm{T}}\\ \Omega_{34} &= P_{13}^{\mathrm{T}} - M_3 + D^{\mathrm{T}}M_4^{\mathrm{T}}\\ \Omega_{35} &= P_{23}^{\mathrm{T}} + M_3C + D^{\mathrm{T}}M_5^{\mathrm{T}}\\ \Omega_{36} &= M_3B + D^{\mathrm{T}}M_6^{\mathrm{T}} - N_3\\ \Omega_{44} &= \tau W + hR + T + rV - M_4 - M_4^{\mathrm{T}}\end{aligned}$$