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A cooperative stochastic differential game of transboundary industrial pollution[☆]

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Abstract

Though cooperation in environmental control holds out the best promise of effective actions, limited success has been observed because existing multinational joint initiatives fail to satisfy the property of subgame consistency. A cooperative solution is subgame consistent if the solution optimality principle is maintained in any subgame which starts at a later time with any feasible state brought about by prior optimal behaviors. This paper presents a cooperative stochastic differential game of transboundary industrial pollution with two novel features. The first feature is that industrial production creates short-term local impacts and long-term global impacts on the environment. Secondly, a subgame consistent cooperative solution is derived in this stochastic differential game together with a payment distribution mechanism that supports the subgame consistent solution. This is the first time that pollution management is analyzed in a cooperative stochastic differential game framework under these novel features.

Keywords: Differential games; Cooperative solution; Subgame consistency; Industrial pollution

1. Introduction

After several decades of rapid technological advances and economic growth, alarming levels of pollution and environmental degradation are emerging all over the world. Due to the geographical diffusion of pollutants, unilateral response on the part of one country or region is often ineffective. Though cooperation in environmental control holds out the best promise of effective action, limited success has been observed. Existing multinational joint initiatives like the Kyoto Protocol or pollution permit trading can hardly be expected to offer a long-term solution because there is no guarantee that participants will always be better off within the entire duration of the agreement. Differential games provide an effective tool to study pollution control problems and to analyze the interactions between the participants' strategic behaviors and dynamic evolution of pollution. Cooperative differential games in environmental control have been presented by Breton, Zaccour, and Zahaf (2005, 2006), Dockner and Long (1993), Fredj, Martín-Herrán, and Zaccour (2004), Jørgensen and Zaccour (2001), Petrosyan and Zaccour (2003), and Yeung (2007).

In the game model developed below, the industrial sector is characterized by an international trading zone involving n nations or regions. Each government adopts its own abatement policy and tax scheme to reduce pollution. The governments have to promote business interests and at the same time have to handle the financing of the costs brought about by pollution. Industrial sectors remain competitive among themselves while governments cooperate in pollution abatement. Most cooperative environmental games do not distinguish the government from the industrial sector in any region. Therefore, international cooperation in environmental

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control involves direct cooperation in industrial production in these analyses.

A special feature of this stochastic differential game model is that industrial production creates two types of negative environmental externalities. First, pollutants emitted via industrial production cause short term local impacts on neighboring areas of the origin of production. Examples of these short term local impacts include passing-by waste in waterways, wind-driven suspended particles in air, unpleasant odour, noise, dust and heat generated in the production processes. Second, the emitted pollutants would add to the existing pollution stock in the environment and produce long term impacts to extensive and far-away areas. Green-house-gas, CFC and atmospheric particulates are examples of this form of negative environmental externality. This specification, which is not found in existing cooperative dynamic environmental games, permits the proximity of the origin of industrial production to receive heavier environmental damages as production increases. Given these neighboring impacts the individual government tax policy has to take into consideration the tax policies of other nations and the intricate effects of these policies on outputs and environmental effects. In particular, while designing tax policies to curtail their outputs governments have to consider the inducement to neighboring nations' output which could cause local negative environmental impacts to themselves.

To incorporate the widely observed uncertainty in nature's capability to replenish the environment, this paper adopts a stochastic pollution stock dynamics and formulates a cooperative stochastic differential game of transboundary industrial pollution. The number of solvable cooperative stochastic differential games so far remains low because of difficulties in deriving tractable solutions (like Haurie, Krawczyk, and Roche (1994) and Yeung and Petrosyan (2004, 2005, 2006a)). A particularly stringent condition – subgame consistency – is required for a dynamically stable cooperative solution in stochastic differential games. A cooperative solution is subgame consistent if the solution optimality principle is maintained in any subgame which starts at a later time with any feasible state brought about by prior optimal behaviors. Since all players are guided by the same optimality principle at each instant of time, they do not possess incentives to deviate from the previously adopted optimal behavior throughout the game. An optimality principle which shares the expected gain from cooperation proportional to the nations' relative sizes of expected noncooperative payoffs is adopted and a payment mechanism which ensures a subgame-consistent solution is explicitly derived.

The paper is organized as follows. Section 2 provides the game formulation. Noncooperative outcomes are characterized in Section 3. Cooperative arrangements, group optimal actions, individual rationality and subgame-consistent imputations are examined in Section 4. A payment distribution mechanism bringing about the proposed subgame-consistent solution is derived in Section 5. A numerical example is provided in Section 6 and concluding remarks are given in Section 7.

2. Game formulation

In this section we present a stochastic differential game model of environmental with n asymmetric nations or regions.

2.1. The industrial sector

Consider a multinational economy which is comprised of n nations. To allow different degrees of substitutability among the nations' outputs a differentiated products oligopoly model has to be adopted. The differentiated oligopoly model used by Dixit (1979) and Singh and Vives (1984) in industrial organizations is adopted to characterize the interactions in this international market. In particular, the nations' outputs may range from a homogeneous product to n unrelated products. Specifically, the demand function of the output of nation $i \in N \equiv \{1, 2, \ldots, n\}$ at time instant s is

$$P_i(s) = \alpha^i - \sum_{j=1}^{n} \beta_j^i q_j(s), \tag{1}$$

where $P_i(s)$ is the price of the output of nation i, $q_j(s)$ is the output of nation j, α^i and β_j^i for $i \in N$ and $j \in N$ are positive constants. The output choice $q_j(s) \in [0, \bar{q}_j]$ is nonnegative and bounded by a maximum output constraint $\bar{q}_j$. Output price equals zero if the right-hand-side of () becomes negative. The demand system (1) shows that the economy is a form of differentiated products oligopoly with substitute goods. In the case when $\alpha^i = \alpha^j$ and $\beta_j^i = \beta_i^j$ for all $i \in N$ and $j \in N$, the industrial outputs resemble is a homogeneous good.In the case when $\beta_j^i = 0$ for $i \neq j$, the n nations produce n unrelated products. Moreover, the industry equilibrium generated by this oligopoly model is computable and fully tractable.

Industrial profits of nation i at time s can be expressed as:

$$\pi_i(s) = \left[\alpha^i - \sum_{j=1}^{n} \beta_j^i q_j(s)\right] q_i(s) - c_i[q_i(s)] - v_i(s) q_i(s),$$
$$\text{for } i \in N, \tag{2}$$

where $v_i(s) \geq 0$ is the tax rate imposed by government i on its industrial output at time s and c_i is the unit cost of production. At each time instant s, the industrial sector of nation $i \in N$ seeks to maximize (2). Note that each industrial sector would consider the information on the demand structure, each other's cost structures and tax policies. The first order condition for a Nash equilibrium for the n nations economy yields

$$\sum_{j=1}^{n} \beta_j^i q_j(s) + \beta_i^i q^i(s) = \alpha^i - c_i - v_i(s), \ \text{for } i \in N. \tag{3}$$

With output tax rates $v(s) = \{v_1(s), v_2(s), \ldots, v_n(s)\}$ being regarded as parameters (3) becomes a system of equations linear in $q(s) = \{q_1(s), q_2(s), \ldots, q_n(s)\}$. Solving (3) yields an industry equilibrium:

$$q_i(s) = \phi_i^i(v(s)) = \bar{\alpha}^i + \sum_{j \in N} \bar{\beta}_j^i v_j(s), \tag{4}$$