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A stochastic control model for optimal timing of climate policies ☆

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Abstract

A stochastic control model is proposed as a paradigm for the design of optimal timing of greenhouse gas (GHG) emission abatement. The resolution of uncertainty concerning climate sensitivity and the technological breakthrough providing access to a carbon-free production economy are modeled as controlled stochastic jump processes. The optimal policy is characterized using the dynamic programming solution to a piecewise deterministic optimal control problem. A numerical illustration is developed with a set of parameters calibrated on recently proposed models for integrated assessment of climate policies. The results are interpreted and the insights they provide on the timing issue of climate policy are discussed.

Keywords: Stochastic control; Piecewise deterministic Markov process; Climate policies; Environmental hedging strategies

1. Introduction

While climate change has become an issue of primary concern for virtually all countries around the world, highly industrialized nations rightly feel particularly concerned. Indeed, it is the strong correlation between wealth production and maintenance of wealthy lifestyles on the one hand, and energy consumption and consequent pollution on the other hand, that makes them the historical culprits for the current state of the planet. This suggests they be considered somewhat more responsible in ensuring its survival, or rather our collective survival in it. Developing industrial giants, or immense consumer pools to be, such as China and India, will no doubt also have to play their part in helping stem the tide, but they can argue that they are relatively recent, albeit potentially very significant, polluters in this game. At the heart of the matter however, lie economic considerations. How can governments then best tread the line between wealth producing activities (typically correlated with energy voraciousness either as a process or as a consequence), and the carrying of their fair share of controlling greenhouse gas (GHG) emissions?

Rational answers to the above question critically depend on both an understanding of the impact of human activity on climate, in particular the impact of GHG concentrations in the earth atmosphere on average surface air temperature (SAT), and on an ability to limit the release of such gases in the atmosphere either via improved technologies, or a switch to less polluting, non-fossil, renewable forms of energy, the latter being typically much more expensive to produce, or difficult to generate. However, precise climate sensitivity information, critical for decision making, is currently clouded with a great degree of uncertainty, and the same applies to the future of our pollution restricting technological know-how, both for intrinsic reasons and because it depends on still unknown economic decisions as to future research investment levels.

The aim of this paper is to contribute an elementary model for environmentally conscious, rational economic decision making under uncertainty, with the long term goal of maintaining as high a level of welfare as permitted by the state of our planet and our technological know-how. A global infinite horizon discounted welfare criterion is specified, and decision making is formulated as an optimal stochastic control

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problem on a jump Markov (rather than diffusion) model. Such models include continuous evolutions labeled as modes, punctuated by mode jumps at random times. The model is construed as representing, albeit in a highly aggregated manner, the evolution of the critical decision parameters as they depend on time and the dynamic levels of capital investment. The model extends preliminary analyses of authors such as Labriet, Loulou, and Kanudia (2008) and Manne (2005) which, although dealing with more realistic models of the economy, rely on stochastic programming methods that can only produce control strategies adapted to the (limited) family of candidate disturbance realizations considered in the computations. By contrast, the approach presented here leads to closed-loop optimal controls.[1] While closed-loop policies are more desirable, the complexity of the associated computations limits the level of realism one can aim for in the modeling of the economy and environment. However, our approach should be viewed as a means of selecting promising candidate control strategies to be validated via simulations on more realistic integrated assessment models of the economy and environment.

The use of optimal economic growth models, in line with the original model of Ramsey (1928) – very much akin to optimal control models – to build integrated assessment models for climate policy has been proposed by many authors such as Filar, Gaertner, and Janssen (1995), Gaertner (2001), Manne, Mendelsohn, and Richels (1995), Manne and Richels (1992, 2005), Nordhaus (1992, 1994) and Nordhaus and Yang (1996), More recently, the use of stochastic control models to develop climate-economy models has been advocated by Ambrosi et al. (2003), Carraro and Filar (1995), Haurie (2003) and Zapert, Gaertner, and Filar (1998).

Jump Markov models are associated by now with a rich control theoretic literature particularly starting with the works of Rishel (1975) and Wonham (1970), followed by Dufour and Costa (1999), Mariton (1990) and Sworder (1969), for example. A generator theory for their non-diffusive version, including the so-called boundary induced or (equivalently) forced jumps, has been developed by Davis in his definitive monograph (Davis, 1993), and leads directly to related dynamic programming equations (see Vermes (1985), for example).

The rest of the paper is organized as follows. In Section 2, we present our integrated assessment model. It includes a continuous part capturing the production process, as well as two distinct types of one-time random mode jumps. The first type of random jump is aimed at modeling the potentially abrupt move from current uncertainty in climatological knowledge, to a state of more thorough knowledge as data are recorded over time (in particular a definitive assessment of the earth mean SAT sensitivity to GHG concentration levels); the second type of jump is aimed at capturing the randomly timed occurrence of the widely anticipated move from the current state of limited environment related technological know-how, to technological breakthroughs leading to carbon-free or lowered carbon production economies. Note that the timing of the occurrence of the second type of jump is strongly affected by cumulative capital investment into research programs. Further building on the model, in Section 3, a welfare criterion is introduced, the nature of control actions is detailed and an optimal control problem with penalty based enforcement of a precautionary principle is proposed. Section 4 is dedicated to the presentation of the associated dynamic programming equations and a discussion relating their solution to that of an intermediary sequence of deterministic infinite horizon optimal control problems. It provides also economic interpretations of the various rewards/decisions in the model. In Section 5, we present how we solve numerically our infinite horizon optimal control problems. Optimal control results are then discussed in Section 6 and simulation results in Section 7. Conclusions are drawn in Section 8.

[1] Note that while observations are made only at jump times, the control law is feedback type and could thus account for continuous observations.

2. The integrated assessment model

2.1. Economic modeling

We use an economic growth model a la Ramsey (1928) where the economic good is produced by three factors, labor, physical capital and fossil energy which generates carbon emissions. We distinguish between two types of economy: the "carbon economy" (our present economy) where a high level of carbon emissions is necessary to obtain output and a so-called "carbon-free economy" (an hydrogen economy, for instance) where a much lower level of emissions is necessary to produce the economic good. As the associated technologies are completely different, we use two distinct types of capital for the two economies. The carbon-free capital cannot initially contribute to the production of economic output. For that to happen, the associated technology must first become available, that is a "breakthrough" is a precondition. However, investment into the carbon-free technology will increase the probability that such a breakthrough takes place. So the carbon-free related physical capital is treated as cumulative research and development (R&D) investment until the anticipated breakthrough occurs, after which it is treated as a productive capital. A good illustration of such a technology behavior is controlled hydrogen fusion. It has attracted significant investments over time with no ability as yet to generate electricity (no economic output). A transition to such a stage[2] would undoubtedly be an abrupt phenomenon as was the case with controlled nuclear fission.

2.1.1. Variables

The following variables enter in the description of the economic model:

$C(t) \geq 0$: total consumption at time t, in trillions (10^{12}) of dollars;
$c(t)$: per capita consumption, $c(t) = \frac{C(t)}{L(t)}$;
$E(t) \geq 0$: global yearly emissions of GHG (in Gt-10^9 tons-carbon equivalent);

[2] Such a breakthrough would usher the way into a carbon-free economy whereby fusion based electricity could partly be used to produce hydrogen in turn to be used as the main fuel throughout the economy.