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Brief paper

Active mode observability of switching linear systems☆

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Abstract

Active mode observability is addressed for a class of discrete-time linear systems that may switch in an unknown and unpredictable way among different modes taken from a finite set. The active mode observation problem consists in determining control sequences (discerning control sequences) that allow to reconstruct the switching sequence on the basis of the observations. The presence of unknown but bounded noises affecting both the system and measurement equations is taken into account. A general condition is derived that characterizes discerning controls in a finite-horizon setting. Such a result is extended to the infinite-horizon case in order to derive "persistently discerning" control sequences. A numerical example is reported to clarify the approach.

Keywords: Active estimation; Linear systems; Mode observability; Switching systems

1. Introduction

Over the past decade, particular attention has been devoted to the study of *hybrid systems*. Such systems result from the interaction of continuous dynamics, discrete dynamics, and logic decisions, and can be used to describe a wide range of physical and engineering systems. For different classes of hybrid systems, specific definitions of observability can be given. For jump linear systems, i.e., systems in which the evolution of the discrete dynamics is governed by Markov processes, the reader is referred to Mariton (1990) for a survey and to Fang and Loparo (2002), Costa and Tuesta (2003) for recent developments. For piece-wise affine systems, where the discrete state is a piece-wise function of the previous continuous state, the reader is referred to Bemporad, Ferrari-Treccate, and Morari (2000), Collins and van Schuppen (2004). Finally, for systems where the control input is extended to the system mode (the system mode is a control variable) the reader is referred to Sun and Ge (2005).

In this paper, the focus is on a particular class of hybrid systems: *switching discrete-time linear systems*, i.e., linear systems in which the matrices governing the system and measurement equations can take values in a finite set (the index denoting such values is called the "mode" or the "discrete state"). We assume the system matrices may switch at each time instant in an unknown and unpredictable way. For such a class of systems, we investigate the connection between the choice of the control sequence and the observability of the system mode. More specifically, the problem we address consists in looking for suitable control sequences such that the switching sequence can be reconstructed on the basis of the observations. We call this problem *active mode observation*, borrowing the term "active" from the literature on active estimation/identification (see Hu & Ersson, 2004; Loparo, Feng, & Fang, 1997; Scardovi, Baglietto, & Parisini, 2007). It is important to remark that the knowledge of the past discrete states may be of interest in many practical situations. A possible application is fault diagnosis, as the discrete state is often introduced to model possible faults in a plant. Moreover, once the discrete state is known, standard filtering techniques can be efficiently applied to estimate the continuous state of the system.

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For unforced noise-free switching systems (where the control is absent), the problem of determining the switching sequence on the basis of the observations was first addressed systematically in Vidal, Chiuso, and Soatto (2002) under the assumption of a minimum dwell time between consecutive switches. More recent advances on this topic have been developed in Babaali and Egerstedt (2004) where arbitrary switching sequences were considered. In Alessandri, Baglietto, and Battistelli (2005), such results have been extended to comply with the presence of bounded disturbances that corrupt the dynamics and the measures. It is important to point out that, for unforced systems, the possibility of exactly reconstructing the sequence of discrete states depends on the initial continuous state of the system. In fact, even in the best-case scenario, it is not possible to determine uniquely the switching sequence when the initial continuous state is null (or, in the noisy case, when it belongs to a non-empty neighborhood of the origin). In Babaali and Egerstedt (2004), it has been shown that for noise-free systems such a drawback may be overcome by means of a suitable choice of the controls, thus making it possible to determine uniquely the switching sequence over a finite horizon on the basis of the observations, regardless of the initial continuous state. Such controls are called *discerning controls*.

The original contributions of this paper can be summarized as follows: (i) in the finite-horizon setting, the results of Babaali and Egerstedt (2004) are extended to the noisy case and a characterization of discerning control sequence in the presence of unknown but bounded noises is derived; (ii) it is proved that, when the mode observability property holds over a finite horizon, then it is possible to derive a *persistently discerning* control sequence over an infinite horizon that, at any time, allows one to reconstruct the discrete state up to the current time; (iii) a sequential scheme is proposed that allows to derive a persistently discerning control sequence by choosing the control vectors on line, one at each time stage.

2. Mode observability over a finite horizon

Let us consider switching discrete-time linear systems described by

$$x_{t+1} = A(\lambda_t)x_t + B(\lambda_t)u_t + w_t, \tag{1a}$$

$$y_t = C(\lambda_t)x_t + v_t, \tag{1b}$$

where $t = 0, 1, \ldots$ is the time instant, $x_t \in \mathbb{R}^n$ is the continuous state vector (the initial state x_0 is unknown), $\lambda_t \in \mathscr{L} \triangleq \{1, 2, \ldots, L\}$ is the discrete state (or "mode" of the system), $u_t \in \mathbb{R}^k$ is the control vector, $w_t \in \mathscr{W} \subset \mathbb{R}^n$ is the system noise vector, $y_t \in \mathbb{R}^m$ is the vector of the measurements, and $v_t \in \mathscr{V} \subset \mathbb{R}^m$ is the measurement noise vector. $A(\lambda)$, $B(\lambda)$, and $C(\lambda)$, $\lambda \in \mathscr{L}$, are $n \times n$, $n \times k$, and $m \times n$ matrices, respectively. We assume the statistics of the vectors x_0, w_t, and v_t as well as the law governing the evolution of the discrete state to be unknown.

In this section, we study the observability of the discrete state of system (1) over a finite horizon of length $N+1$. More specifically, we want to investigate whether it is possible to choose the control sequence[1] $\mathbf{u}_{0,N-1}$ in such a way that the switching sequence $\boldsymbol{\lambda}_{0,N}$ (or at least a portion of it) can be reconstructed on the basis of the observation sequence $\mathbf{y}_{0,N}$ and of the control sequence $\mathbf{u}_{0,N-1}$ for any possible initial continuous state and any noise sequence. Since the law governing the evolution of the discrete state is supposed to be completely unknown, the switching sequence $\boldsymbol{\lambda}_{0,N}$ may take on any value in the set $\mathscr{L}^{N+1}$.

In order to address the active mode observation problem some preliminary definitions are useful. First note that the observation sequence $\mathbf{y}_{0,N}$ can be written as

$$\mathbf{y}_{0,N} = F(\boldsymbol{\lambda}_{0,N})x_0 + G(\boldsymbol{\lambda}_{0,N})\mathbf{u}_{0,N-1} + H(\boldsymbol{\lambda}_{0,N})\mathbf{w}_{0,N-1} + \mathbf{v}_{0,N},$$

where $F(\boldsymbol{\lambda}_{0,N})$ is the observability matrix and $G(\boldsymbol{\lambda}_{0,N}), H(\boldsymbol{\lambda}_{0,N})$ are suitable matrices (see Alessandri et al., 2005). Furthermore, let us denote by $\mathscr{S}(\boldsymbol{\lambda}_{0,N}, \mathbf{u}_{0,N-1})$ the set of all the possible observation sequences associated with the switching sequence $\boldsymbol{\lambda}_{0,N}$ and the control sequence $\mathbf{u}_{0,N-1}$ for any possible initial continuous state and any possible noise sequence, i.e.,

$$\begin{aligned}\mathscr{S}(\boldsymbol{\lambda}_{0,N}, \mathbf{u}_{0,N-1}) \triangleq \{\mathbf{y} \in \mathbb{R}^{m(N+1)} : \mathbf{y} = F(\boldsymbol{\lambda}_{0,N})x \\ + G(\boldsymbol{\lambda}_{0,N})\mathbf{u}_{0,N-1} + H(\boldsymbol{\lambda}_{0,N})\mathbf{w} + \mathbf{v}, \\ x \in \mathbb{R}^n, \mathbf{w} \in \mathscr{W}^N, \mathbf{v} \in \mathscr{V}^{N+1}\}.\end{aligned}$$

As the observation sequence $\mathbf{y}_{0,N}$ depends on the control sequence $\mathbf{u}_{0,N-1}$, one may think of suitably choosing $\mathbf{u}_{0,N-1}$ in order to make it possible to uniquely determine the switching sequence $\boldsymbol{\lambda}_{0,N}$ or, at least, a portion of it. More specifically, in the lines of Alessandri et al. (2005), we shall look for two integers α and ω, with $\alpha, \omega \geqslant 0$ and $\alpha + \omega \leqslant N$, such that it is possible to uniquely determine the discrete state λ_t in the restricted interval $[\alpha, N - \omega]$ on the basis of the observation sequence $\mathbf{y}_{0,N}$ and of the control sequence $\mathbf{u}_{0,N-1}$. Towards this end, given a switching sequence $\boldsymbol{\lambda}$ in the interval $[0, N]$, let us denote by $r^{\alpha,\omega}(\boldsymbol{\lambda})$ the restriction of $\boldsymbol{\lambda}$ to the interval $[\alpha, N - \omega]$. Of course, from a practical point of view, it would be preferable to have $\alpha = 0$ and $\omega = 0$; unfortunately, in some cases, reconstructing the entire sequence $\boldsymbol{\lambda}_{0,N}$ turns out to be an impossible task (see, for instance, the example system of Section 4). Note that a similar idea was proposed in Babaali and Egerstedt (2004), where only the case $\alpha = 0$ and $\omega \geqslant 0$ was considered. Here, the presence of α introduces a further degree of freedom in the mode observation scheme. We are now ready to give the following definition.

Definition 1. System (1) is said to be (N, α, ω)-*mode observable* if there exists a control sequence $\mathbf{u}_{0,N-1}$ such that, for every pair of switching sequences $\boldsymbol{\lambda}, \boldsymbol{\lambda}' \in \mathscr{L}^{N+1}$ with $r^{\alpha,\omega}(\boldsymbol{\lambda}) \neq r^{\alpha,\omega}(\boldsymbol{\lambda}')$, we have

$$\mathscr{S}(\boldsymbol{\lambda}, \mathbf{u}_{0,N-1}) \bigcap \mathscr{S}(\boldsymbol{\lambda}', \mathbf{u}_{0,N-1}) = \emptyset.$$

A control sequence $\mathbf{u}_{0,N-1}$ that has such a property is called an (N, α, ω)-*discerning control sequence* for system (1).

[1] Given a generic sequence $\mathbf{z}_{0,\infty} \triangleq \{z_t; t=0, 1, \ldots\}$ and two time instants $t_1 \leqslant t_2$, we define $\mathbf{z}_{t_1,t_2} \triangleq \mathrm{col}\,(z_{t_1}, z_{t_1+1}, \ldots, z_{t_2})$.