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ScienceDirect
Automatica 44 (2008) 1426–1430
automatica
www.elsevier.com/locate/automatica

Technical communique

A nonlinear observer for on-line estimation of the cerebrospinal fluid outflow resistance☆

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Received 30 June 2006; received in revised form 1 February 2007; accepted 12 September 2007
Available online 3 March 2008

Abstract

Accurate estimates of the outflow resistance of the human cerebrospinal fluid system are important for the diagnosis of a medical condition known as hydrocephalus. In this paper we design a nonlinear observer which provides on-line estimates of the outflow resistance, to the best of our knowledge the first method to do so. The output of the observer is proven to globally converge to an unbiased estimate. Its performance is experimentally verified using the same apparatus used to perform actual patient diagnoses and a specially-designed physical model of the human cerebrospinal fluid system.

Keywords: Nonlinear observers; Biomedical systems; Parameter estimation

1. Introduction

Estimation of the outflow resistance of the cerebrospinal fluid (CSF) is an important problem in medical diagnosis. Patients with a condition known as Idiopathic Normal Pressure Hydrocephalus (INPH), a form of communicating hydrocephalus, are observed to have a disturbance to the hydrodynamics of the CSF system (Malm & Eklund, 2006).

It is well established that the symptoms of INPH can be reduced or eliminated by implanting a shunt, a small valve with an attached drainage tube, into the patient's skull, thereby altering the system and increasing the CSF flow (Bergsneider, Black, Klinge, Marmarou, & Relkin, 2005).

The disease is difficult to diagnose and any such surgery has potential risks, so neurologists and neurosurgeons use a number of different diagnostic tests to decide whether or not the patient is likely to benefit from a shunt surgery. Estimation of the outflow resistance is one of the tests currently used, and higher than average resistance values are suggested to be an indicator for shunt surgery (Malm & Eklund, 2006).

The value of the resistance is not directly measurable, but must be inferred from measurements of fluid pressure, and an assumed hydrodynamical model. In this paper we propose a nonlinear observer which provides on-line estimates of the outflow resistance, to our knowledge the first method to do so.

All resistance estimation methods in current clinical use rely on off-line analysis of data (for examples, see Andersson, Malm, Backlund, and Eklund (2005), Czosnyka et al. (1990) and Marmarou, Shulman, and Rosende (1978)). Typically, artificial CSF is injected into the spinal column with a particular flow pattern. The resulting pressure variations are recorded. Following the experiment, certain standard formulas are applied, usually to the portions of the data in which the system has reached a steady state, and a resistance estimate is obtained.

A challenge with off-line estimation is that the physiological disturbances due to vasogenic blood volume variations in the craniospinal system are often large in comparison to the net

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Mayuresh Kothare under the direction of Editor André Tits.

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0005-1098/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2007.09.025

external inflow and CSF formation rate. The magnitude of such variations differs a lot between patients. This makes it difficult to choose an appropriate testing period in advance because patients with larger flow variations will require longer tests to achieve a reliable estimate.

Minimizing the test time benefits both the patient, who must endure shorter periods of discomfort, and the hospital, requiring less time of trained nursing staff. An on-line test allows the staff to monitor the progress of the estimated resistance, and end the test when it has converged to a stable value. This can allow shorter tests for patients with small flow variations, whilst ensuring sufficient data has been collected for those with large flow variations.

2. The cerebrospinal fluid system

Cerebrospinal fluid is generated by the body and introduced to the brain cavity at an approximately constant rate (Ekstedt, 1978). It provides physical support for the brain, and is believed to absorb and carry away toxic metabolic byproducts. It leaves the brain cavity by being reabsorbed into the bloodstream in the dural sinuses (Fishman, 1992).

The CSF system is an infinite-dimensional distributed parameter system, however it has been shown experimentally to be well approximated by a finite-dimensional lumped-parameter model in which the intracranial pressure, hereafter denoted $P_{ic}(t)$, and other variables are assumed to be spatially invariant (Czosnyka, Czosnyka, Momjian, & Pickard, 2004; Marmarou, Shulman, & LaMorgese, 1975; Marmarou et al., 1978; Sivaloganathan, Tenti, & Drake, 1998). In this model, the flow out of the intracranial cavity due to absorption into the bloodstream, I_a, is proportional to the difference between P_{ic} and the pressure in the dural sinuses, P_{ds}:

$$I_a(t) = (P_{ic}(t) - P_{ds})/R. \tag{1}$$

It is the outflow resistance, R, which we wish to estimate. The CSF fluid is essentially incompressible, so a simple conservation of volume equation can be proposed:

$$\frac{\mathrm{d}}{\mathrm{d}t} V_{\mathrm{CSF}}(t) = I_f + I_{\mathrm{ext}}(t) - I_a(t), \tag{2}$$

where $V_{\mathrm{CSF}}(t)$ is the volume of CSF inside the brain cavity, I_f is the formation rate of CSF produced by the body, which is assumed constant (Ekstedt, 1978), and $I_{\mathrm{ext}}(t)$ is the external inflow of artificial CSF introduced during the experiment.

The pressure and volume of the brain cavity are related by a pressure-dependent compliance term $f_c(P_{ic}(t))$ like so:

$$f_c(P_{ic}) \frac{\mathrm{d}}{\mathrm{d}t} P_{ic}(t) = \frac{\mathrm{d}}{\mathrm{d}t} V_{\mathrm{CSF}}(t). \tag{3}$$

In this paper a compliance function of the following form is assumed:

$$f_c(P_{ic}) \coloneqq 1/(kP_{ic}), \tag{4}$$

where k is a positive constant, which varies from person to person. This form of compliance is common in the literature, and there exists experimental procedures to determine the value of k, such as the bolus infusion test (Marmarou et al., 1978). In this paper we assume it is available.

We now introduce a new constant, the resting pressure, denoted P_r. It is the steady state pressure reached due to I_f only:

$$P_r = I_f R + P_{ds}. \tag{5}$$

We cannot measure P_{ds}, however P_r is available, being the value of $P_{ic}(t)$ when $I_{\mathrm{ext}}(t) = 0$.

Now, substituting (1), (2), (4) and (5) into (3) we obtain the following nonlinear differential equation for $P_{ic}(t)$:

$$\frac{\mathrm{d}}{\mathrm{d}t} P_{ic}(t) = -\frac{k}{R} [P_{ic}(t)]^2 + \left(k I_{\mathrm{ext}}(t) + \frac{k P_r}{R} \right) P_{ic}(t). \tag{6}$$

This is the model we will use to design our observer. In the design we will make use of the fact that the change of variables $x(t) = 1/P(t)$ and $\theta = 1/R$ results in the following equation:

$$\dot{x}(t) = -\dot{P}_{ic}(t)/P_{ic}^2(t) = k\theta - (k I_{\mathrm{ext}}(t) + k P_r \theta)\, x(t). \tag{7}$$

3. The observer

The objective is to obtain an estimate of the constant R in real-time, from measurements of the signals $P_{ic}(t)$ and $I_{\mathrm{ext}}(t)$ and knowledge of the constants P_r and k. We propose a nonlinear observer which generates coupled estimates of $P_{ic}(t)$ and R based on the system (6). It is defined by the following equations[1]

$$\frac{\mathrm{d}}{\mathrm{d}t} \hat{x} = -k I_{\mathrm{ext}} \hat{x} + \hat{\theta} \left(k - \frac{k P_r}{P_{ic}} \right) + c \left(\hat{x} - \frac{1}{P_{ic}} \right), \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \hat{\theta} = -\gamma \left(k - \frac{k P_r}{P_{ic}} \right) \left(\hat{x} - \frac{1}{P_{ic}} \right), \tag{9}$$

$$\hat{P}_{ic}(t) = 1/\hat{x}(t), \qquad \hat{R}(t) = 1/\hat{\theta}(t). \tag{10}$$

The gains γ and c can be adjusted to tune convergence rates, and must satisfy the following inequalities:

$$\gamma > 0, \quad c < k I_{\mathrm{ext}}(t) - \delta \quad \forall\, t \geq 0, \tag{11}$$

for some constant $\delta > 0$.

Remark 1. It is impossible for $P_{ic}(t)$ and R to be anything but positive numbers. In Section 4 it will be proved that $\hat{x}(t)$ and $\hat{\theta}(t)$ converge globally and asymptotically to $1/P_{ic}(t)$ and $1/R$, respectively, and hence $\hat{P}_{ic}(t)$ and $\hat{R}(t)$ will eventually be well defined. Although it is possible for $\hat{x}(t)$ or $\hat{\theta}(t)$ to cross zero early in the experiment, the experimenter can disregard these values as nonsensical, and wait for the convergence to take its course.

In connection with this observer, we make the following definition:

[1] For the sake of brevity of expression, we will occasionally drop the (t) arguments from signals.