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Brief paper

MIMO controller synthesis with integral-action integrity ☆

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Abstract

A controller synthesis method is presented for closed-loop stability and asymptotic tracking of step input references with zero steady-state error. Integral-action is achieved in two design steps starting with any stabilizing controller and adding a PID-controller in a configuration that guarantees robust stability and tracking. The proposed design has integral-action integrity, where closed-loop stability is maintained even when any of the proportional, integral, or derivative terms are removed or the entire PID-controller is limited by a constant gain matrix. The integral constant can be switched off when integral-action is not wanted.

Keywords: Integral action; PID controllers; Steady-state tracking

1. Introduction

We consider integral-action controller design for linear, time-invariant (LTI) multi-input multi-output (MIMO) plants. Our goal is to achieve closed-loop stability and robust asymptotic tracking of step-input references with zero steady-state error. This objective is extended to type-m integral action in each output channel so that polynomial references up to order $m-1$ applied at each input would be asymptotically tracked.

The simplest controllers that achieve integral-action are in the proportional+integral+derivative (PID) form. However, closed-loop stability can be achieved using these low order controllers only for certain classes of plants, and many others cannot be stabilized using PID-controllers (Gündeş & Wai, 2005). The standard method of achieving integral-action is the well-known full-order observer-based integral-action controller design based on an augmented plant model, which uses linear quadratic regulator (LQR) or pole-placement methods to find state-feedback gains for the states of the integrators in addition to the states of the plant (Goodwin, Graebe, & Salgado, 2001). Although this method achieves both closed-loop stability and steady-state accuracy, the integrators cannot be completely switched off without affecting closed-loop stability. Furthermore, this standard method does not easily extend to higher integral-action type (Gündeş & Kabuli, 1998). In this paper we propose a two-step integral-action synthesis procedure that achieves robust tracking by adding a PID-controller over a previously designed stabilizing controller that is already present in the feedback loop. An initial stabilizing controller, which does not have integral-action, is designed (to be optimal and to satisfy given design objectives) for the original plant using any method (LQR, H_∞, etc.). Then an additional PID-controller is designed for a stable system (the numerator-matrix of the plant). The two controllers are then configured to achieve closed-loop stability and integral-action together. All integral-action controllers can be obtained by inclusion of a free controller parameter. The main advantage of this two-step approach is that the PID-controller block containing the integral-action designed in the second step can be switched off (taken out completely and the states are reset) without affecting closed-loop stability. The PID-controller can be designed with an additional property that we call integral-action integrity, where closed-loop stability is maintained even when any of the proportional, integral, or derivative terms are removed or the entire PID block is limited by a constant gain

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matrix. If the design requires a higher or lower integral-action type, the initial design can be easily modified by including incrementally designed additional integrator terms in the controller. High-order integral terms can be deleted to achieve a lower type all without re-designing the entire stabilization loop. This incremental feature of the design starting from stabilizing controllers for the original plant and adding on integrators as necessary makes it possible to compare the system performance for different integral-action types since all designs are based on the original plant instead of different augmented systems. In contrast with the standard approach to integral-action design for an augmented system, the design proposed here does not use an augmented system identification and does not need to re-identify the plant for a stabilizing controller without that integral-action component or a lower/higher-order integral-action component. Simulation comparisons of the proposed method with the standard augmentation-based method were given for a stable plant in Mete and Gündeş (2004). Since the performance of integral-action control depends on the system operating in a linear range and integral-action controllers suffer serious loss of performance due to integral windup, which occurs when the actuators in the control-loop saturate, it may be desirable to switch off the integral term while maintaining closed-loop stability for protection against windup (Doyle, Smith, & Enns, 1987; Kothare, Campo, Morari, & Nett, 1994; Kapoor, Teel, & Daoutidis, 1998). The methods proposed here simply design controllers whose integral-action components can be turned-off (or limited), and are not intended as alternate anti-windup schemes. When the integral-action is turned-off, the states in the part of the controller implementation that is taken out of service are all set to zero and the initial conditions and outputs are reset to zero.

Although continuous-time systems are discussed, all results apply also to discrete-time systems with appropriate modifications. *Notation*: $\mathcal{U}$ is the extended closed right-half plane, i.e., $\mathcal{U} = \{s \in \mathbb{C} \,|\, \mathcal{R}e(s) \geqslant 0\} \cup \{\infty\}$; $\mathbb{R}$, $\mathbb{R}_+$ denote real and positive real numbers; $\mathbf{R_p}$ denotes real proper rational functions of s; $\mathbf{S} \subset \mathbf{R_p}$ is the stable subset with no poles in $\mathcal{U}$; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$; I_n is the $n \times n$ identity matrix; we use I when the dimension is unambiguous. The H_∞-norm of $M(s) \in \mathcal{M}(\mathbf{S})$ is denoted by $\|M(s)\|$ (i.e., the norm $\|\cdot\|$ is defined as $\|M\| := \sup_{s\in\partial\mathcal{U}} \bar{\sigma}(M(s))$, where $\bar{\sigma}$ is the maximum singular value and $\partial\mathcal{U}$ is the boundary of $\mathcal{U}$). For simplicity, we drop (s) in transfer matrices such as $G(s)$. We use coprime factorizations over $\mathbf{S}$; i.e., for $G \in \mathbf{R_p}^{r\times q}$, $G = XY^{-1}$ denotes a right-coprime-factorization (RCF), $G = \widetilde{Y}^{-1}\widetilde{X}$denotes a left-coprime-factorization (LCF), where $X, \widetilde{X} \in \mathbf{S}^{r\times q}$, $Y \in \mathbf{S}^{q\times q}$, $\widetilde{Y} \in \mathbf{S}^{r\times r}$, $\det Y(\infty) \neq 0$, $\det \widetilde{Y}(\infty) \neq 0$.

2. Problem description and preliminaries

Consider the LTI, MIMO unity-feedback system $Sys(G, \widehat{C})$ in Fig. 1; $G \in \mathbf{R_p}^{r\times q}$ and $\widehat{C} \in \mathbf{R_p}^{q\times r}$ denote the plant's and the controller's transfer-functions. It is assumed that $Sys(G, \widehat{C})$ is well-posed, G and $\widehat{C}$ have no unstable hidden-modes, and $G \in \mathbf{R_p}^{r\times q}$ is full normal rank. Let $H_{eu} = (I_r + G\widehat{C})^{-1} = I_r - G\widehat{C}(I_r + G\widehat{C})^{-1} =: I_r - GH_{wu} =: I_r - H_{yu}$ denote the (input-error) transfer-function from u to e.

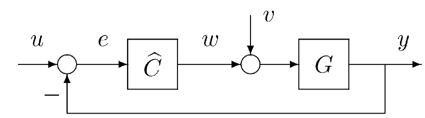

Fig. 1. Unity-feedback system $Sys(G, \widehat{C})$.

Definition 1. (i) The system $Sys(G, \widehat{C})$ is called stable iff the closed-loop transfer-function from (u, v) to (y, w) is stable. (ii) The controller $\widehat{C}$ stabilizes G iff $\widehat{C}$ is proper and $Sys(G, \widehat{C})$ is stable. (iii) The stable system $Sys(G, \widehat{C})$ has integral-action iff H_{eu} has blocking-zeros at $s = 0$; it has type-m integral action in each output channel iff H_{eu} has (at least) m blocking-zeros at zero, i.e., $(s^{-(m-1)}H_{eu})(0) = 0$. (iv) The controller $\widehat{C}$ is called a controller with integral-action iff $\widehat{C}$ stabilizes G and D_c of any RCF $\widehat{C} = N_c D_c^{-1}$ has blocking-zeros at $s = 0$, i.e., $D_c(0) = 0$; $\widehat{C}$ is called a controller with type-m integral action iff $\widehat{C}$ stabilizes G and D_c has (at least) m blocking-zeros at $s = 0$, i.e., $(s^{-(m-1)}D_c)(0) = 0$.

Let $G = XY^{-1} = \widetilde{Y}^{-1}\widetilde{X}$ be any RCF, LCF of the plant, $\widehat{C} = N_c D_c^{-1} = \widetilde{D}_c^{-1}\widetilde{N}_c$ be any RCF, LCF of the controller. Then $\widehat{C}$ stabilizes G if and only if $M_L := \widetilde{Y}D_c + \widetilde{X}N_c$ is unimodular, equivalently, $M_R := \widetilde{D}_c Y + \widetilde{N}_c X$ is unimodular (Gündeş & Desoer, 1990; Vidyasagar, 1985). Suppose that $Sys(G, \widehat{C})$ is stable. Then the error $e(t)$ due to step inputs $u(t)$ goes to zero as $t \to \infty$ if and only if $H_{eu}(0) = 0$. Therefore, the stable system $Sys(G, \widehat{C})$ achieves asymptotic tracking of constant reference inputs with zero steady-state error if and only if it has integral-action; it achieves asymptotic tracking of polynomial references up to order $m - 1$iff it has (at least) type-m integral action (León de la Barra, Emami-Naeini, & Chincón, 1998). Write $H_{eu} = (I + G\widehat{C})^{-1} = I - G\widehat{C}(I + G\widehat{C})^{-1} = D_c\, M_L^{-1}\widetilde{Y} = I - XM_R^{-1}\widetilde{N}_c$. By Definition 1, $Sys(G, \widehat{C})$ has integral-action if and only if $H_{eu}(0) = (D_c M_L^{-1}\widetilde{Y})(0) = 0$. If $\widehat{C} = N_c D_c^{-1}$ is an integral-action controller, then $Sys(G, \widehat{C})$ has integral-action. For $H_{eu}(0) = (D_c M_L^{-1}\widetilde{Y})(0) = 0$, it is sufficient but not necessary to have $D_c(0) = 0$. For plants that have poles at $s = 0$, rank $\widetilde{Y}(0) < r$ and hence, the system may achieve integral-action even if $D_c(0) \neq 0$. For plants with no poles at $s = 0$,rank $\widetilde{Y}(0) = r$ implies $Sys(G, \widehat{C})$ has integral-action if and only if $\widehat{C} = N_c D_c^{-1}$ is an integral-action controller, i.e., $D_c(0) = 0$.

Lemma 2.1 states two necessary conditions for integral-action. In Lemma 2.2, stabilizing controllers are decomposed into a sum of two components. A controller designed to stabilize the stable numerator X of the plant G can be added through a denominator factor to any controller that stabilizes G:

Lemma 2.1 (*Necessary conditions for integral-action*). *Let $G \in \mathbf{R_p}^{r\times q}$. If the system $Sys(G, \widehat{C})$ has integral-action, then* (i) (*normal*) rank $G = r \leqslant m$; (ii) *G has no transmission-zeros at $s = 0$.*