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Brief paper

A parametric programming approach to moving-horizon state estimation☆

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Abstract

We propose a solution to moving-horizon state estimation that incorporates inequality constraints in both a systematic and computationally efficient way, akin to Kalman filtering. The proposed method allows the on-line constrained optimization problem involved in moving-horizon state estimation to be solved offline, requiring only a look-up table and simple function evaluations for real-time implementation. The method is illustrated via simulations on a system that has been studied in literature.
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Keywords: Moving horizon; State estimation; Parametric programming; Kalman filter; Constraints

1. Introduction

For linear systems with Gaussian noise, the celebrated Kalman filter (Kalman, 1960) provides a recursive solution to the real-time minimum-variance state estimation problem, given prior knowledge of the distributions of the initial states, disturbances, and measurement noise. The Kalman filter has also been applied to nonlinear systems in the form of the extended Kalman filter (EKF), which is based on linearization of the nonlinear model around the current mean and covariance estimates. However, the EKF may exhibit poor convergence properties (Haseltine & Rawlings, 2005; Maybeck, 1982).

Inspired by the success of real-time optimization over a moving horizon used in model-predictive control (MPC), moving-horizon estimation (MHE) via real-time optimization was suggested as a practical method for addressing model nonlinearities and inequality constraints in state estimation, while keeping the size of the real-time optimization problem finite (Muske, Rawlings, & Lee, 1993; Rao & Rawlings, 2002; Rao, Rawlings, & Lee, 2001; Robertson, Lee, & Rawlings, 1996; Simon & Simon, 2003). Furthermore, it was shown that constrained state estimators have additional useful properties, such as producing unbiased state estimates and smaller state error covariance (Simon & Simon, 2003). Including inequality constraints in MHE provides a mechanism to improve the estimation based on process knowledge (e.g., flow rates or compositions must be greater than or equal to zero), and can also help compensate for poor choices in the prior distributions.

While the power of MHE has been clearly demonstrated, the computational requirements of real-time constrained optimization that MHE entails may render it impractical in cases where computing power is limited and/or data sampling rates are excessive. For example, in real-time monitoring and diagnostics for aircraft engines—a case where Kalman filtering is widespread and constraints on estimates may be known a priori—data are typically collected at rates over tens of Hz, leaving little time for state estimation via real-time optimization (Simon & Simon, 2003).

To address the computational efficiency issues posed by MHE, we propose in this paper an MHE approach that bypasses real-time optimization. At each time point, the proposed algorithm uses input and output data to consult an off-line constructed look-up table that indicates what (Kalman-filter-like) closed-form expression from a finite collection must be used to calculate the state estimate. Both the look-up table and the finite collection of closed-form expressions for state estimation

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are constructed once via off-line optimization. Our approach parallels the multi-parametric programming approach proposed for constrained MPC of linear systems (Pistikopoulos, Dua, Bozinis, Bemporad, & Morari, 2002).

In the rest of the paper, we briefly present elements of Kalman filtering, MHE, and mp-QP that are relevant to this work. Next, we show how mp-QP can be applied to MHE. Finally, we demonstrate the applicability of the proposed approach by presenting simulations on a system that has appeared in literature, and discuss future developments.

2. Background

2.1. System description

Let a dynamic system be described by the discrete-time model

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k), \tag{1}$$

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k, \tag{2}$$

where the time point k takes integer values; $\mathbf{x}_k \in \Re^n$ is the state vector; $\mathbf{u}_k \in \Re^{n_u}$ is the known input vector; the vectors $\mathbf{w}_k \in \Re^{n_w}$ and $\mathbf{v}_k \in \Re^{n_v}$ are random variables (often assumed to be independent and Gaussian) representing disturbances on the state and measured output, respectively; $\mathbf{y}_k \in \Re^{n_y}$ is the measured output vector; the vector functions $\mathbf{f}_k(\cdot) : \Re^n \times \Re^{n_u} \times \Re^{n_w} \to \Re^n$ and $\mathbf{h}_k(\cdot) : \Re^n \to \Re^{n_y}$ represent the discretized form of a set of ordinary or partial differential equations and may not be available in closed form. It is also assumed that the states and disturbances satisfy the following inequality constraints:

$$\mathbf{D_x x}_k \leqslant \mathbf{d_x}, \quad \mathbf{D_w w}_k \leqslant \mathbf{d_w}, \quad \mathbf{D_v v}_k \leqslant \mathbf{d_v}, \tag{3}$$

where the matrices $\mathbf{D_x}$, $\mathbf{D_w}$, $\mathbf{D_v}$ and vectors $\mathbf{d_x}$, $\mathbf{d_w}$, $\mathbf{d_v}$ are known. The origin is assumed to satisfy the constraints on $\mathbf{w}_k$ and $\mathbf{v}_k$. Typically, the constraints take the form of upper and lower bounds on the vector components (e.g., $x_{k,i}^{\min} \leqslant x_{k,i} \leqslant x_{k,i}^{\max}$, where the subscript i refers to the ith component of the vector $\mathbf{x}_k$). It has been shown (Robertson & Lee, 2002) that constraints also allow one to incorporate non-Gaussian distributions (e.g., asymmetric or truncated distributions) in the state estimation problem.

2.2. Moving-horizon state estimation and Kalman Filtering

MHE is performed by solving the following optimization problem in real time at each discrete time point t:

$$\{\hat{\mathbf{x}}_{t-N+1|t}^{mh}, \ldots, \hat{\mathbf{x}}_{t|t}^{mh}, \hat{\mathbf{w}}_{t-N+1|t}^{mh}, \ldots, \hat{\mathbf{w}}_{t-1|t}^{mh}, \hat{\mathbf{v}}_{t-N+1|t}^{mh}, \ldots, \hat{\mathbf{v}}_{t|t}^{mh}\} = \underset{\substack{\hat{\mathbf{x}}_{t-N+1|t},\ldots,\hat{\mathbf{x}}_{t|t},\\ \mathbf{w}_{t-N+1|t},\ldots,\hat{\mathbf{w}}_{t-1|t},\\ \hat{\mathbf{v}}_{t-N+1|t},\ldots,\hat{\mathbf{v}}_{t|t}}}{\operatorname{argmin}} \times \left[\begin{array}{l} \sum_{k=t-N+1}^{t} \|\hat{\mathbf{v}}_{k|t}\|_{\mathbf{R}^{-1}}^2 + \sum_{k=t-N+1}^{t-1} \|\hat{\mathbf{w}}_{k|t} - \bar{\mathbf{w}}\|_{\mathbf{Q}^{-1}}^2 \\ + \|\hat{\mathbf{x}}_{t-N+1|t} - \bar{\mathbf{x}}_{t-N+1|t-N}\|_{\mathbf{P}^{-1}}^2 \end{array} \right] \tag{4}$$

subject to:

$$\hat{\mathbf{x}}_{k+1|t} = \mathbf{f}_k(\hat{\mathbf{x}}_{k|t}, \mathbf{u}_k, \hat{\mathbf{w}}_{k|t}), \quad k = t-N+1, \ldots, t-1, \tag{5}$$

$$\mathbf{y}_k = \mathbf{h}_k(\hat{\mathbf{x}}_{k|t}) + \hat{\mathbf{v}}_{k|t}, \quad k = t-N+1, \ldots, t, \tag{6}$$

$$\mathbf{D_x}\hat{\mathbf{x}}_{k|t} \leqslant \mathbf{d_x}, \quad \mathbf{D_v}\hat{\mathbf{v}}_{k|t} \leqslant \mathbf{d_v}, \quad k = t-N+1, \ldots, t, \tag{7}$$

$$\mathbf{D_w}\hat{\mathbf{w}}_{k|t} \leqslant \mathbf{d_w}, \quad k = t-N+1, \ldots, t-1, \tag{8}$$

where $\|\mathbf{v}\|_{\mathbf{A}}^2 \hat{=} \mathbf{v}^{\mathrm{T}}\mathbf{A}\mathbf{v}$; $\hat{\mathbf{s}}_{k|t}$ denotes the estimated value of $\mathbf{s}$ at time point k, given measurements up to and including time t; and overbar denotes mean value. The positive definite matrices $\mathbf{Q} \succ 0$, $\mathbf{R} \succ 0$, and $\mathbf{P}_{t-N+1} \succ 0$ are the covariances of the state disturbance, output disturbance, and state, respectively. The resulting $\hat{\mathbf{x}}_{t|t}^{mh}$ is the optimal filtered state estimate, while $\{\hat{\mathbf{x}}_{j|t}^{mh}\}_{j=t-N+1}^{t-1}$ are the optimal smoothed estimates of past states. Note that this formulation allows for a non-zero-mean state disturbance, $\bar{\mathbf{w}}$, and assumes a zero-mean output disturbance.

A key issue for MHE is how to update or propagate $(\bar{\mathbf{x}}_{t-N+1|t-N}, \mathbf{P}_{t-N+1})$. The pair $(\bar{\mathbf{x}}_{t-N+1|t-N}, \mathbf{P}_{t-N+1})$ summarizes the model-based influence of old data not explicitly considered in the current finite horizon, i.e., data corresponding to time points before $t - N + 1$. This is the estimation problem counterpart of the terminal cost in receding horizon MPC (Mayne, Rawlings, Rao, & Scokaert, 2000; Nikolaou, 2001; Rawlings, 2000). In probabilistic terms (Rao et al., 2001; Robertson et al., 1996) this is a problem of updating the conditional density function of the state. Unfortunately, it is not generally possible to derive analytical expressions for the conditional density of nonlinear or constrained systems. As a result, the Kalman filter (or EKF) is typically used to approximate $(\bar{\mathbf{x}}_{t-N+1|t-N}, \mathbf{P}_{t-N+1})$ in MHE. A potential problem with using the Kalman filter approximation in MHE is that a poor initial state covariance matrix, $\mathbf{P}_0$, may lead to instability as a result of overweighting past data relative to newer data. Convergence and stability issues have been analyzed (Rao et al., 2001) via dynamic programming and the concept of arrival cost (the analog of cost-to-go in dynamic programming for the control problem). Stability considerations place an upper bound on the arrival cost, which in turn places a lower bound on the initial covariance matrix $\mathbf{P}_0$.

For a linear time-invariant system, model equations (5) and (6) can be written in the following form:

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{B}\mathbf{u}_k + \mathbf{G}\hat{\mathbf{w}}_k, \tag{9}$$

$$\mathbf{y}_k = \mathbf{C}\hat{\mathbf{x}}_k + \hat{\mathbf{v}}_k, \tag{10}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{G}$ are constant matrices.

For a linear time-invariant system without inequality constraints and with horizon extending from the initial time 0 to current time t, the optimization problem to solve for state estimation at time t is

$$\{\hat{\mathbf{x}}_{0|t}^{mh}, \ldots, \hat{\mathbf{x}}_{t|t}^{mh}, \hat{\mathbf{w}}_{0|t}^{mh}, \ldots, \hat{\mathbf{w}}_{t-1|t}^{mh}, \hat{\mathbf{v}}_{0|t}^{mh}, \ldots, \hat{\mathbf{v}}_{t|t}^{mh}\} = \underset{\substack{\hat{\mathbf{x}}_{0|t},\hat{\mathbf{x}}_{t|t},\\ \hat{\mathbf{w}}_{0|t},\ldots,\hat{\mathbf{w}}_{t-1|t},\\ \hat{\mathbf{v}}_{0|t},\ldots,\hat{\mathbf{v}}_{t|t}}}{\operatorname{argmin}} \left[\begin{array}{l} \sum_{k=0}^{t} \|\hat{\mathbf{v}}_{k|t}\|_{\mathbf{R}^{-1}}^2 + \sum_{k=0}^{t-1} \|\hat{\mathbf{w}}_{k|t} - \bar{\mathbf{w}}\|_{\mathbf{Q}^{-1}}^2 \\ + \|\hat{\mathbf{x}}_{0|t} - \bar{\mathbf{x}}_0\|_{\mathbf{P}_0^{-1}}^2 \end{array} \right] \tag{11}$$

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