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Brief paper

# Computer control under time-varying sampling period: An LMI gridding approach ☆

Antonio Sala*

*Department of Systems Engineering and Control – DISA, Universidad Politécnica de Valencia, Camino de Vera, 14, E-46022 Valencia, Spain*

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**Abstract**

This paper addresses computer control under time-varying sampling period and delayed actuation. The proposed approach uses time-varying observers and state-feedback controllers designed by means of linear matrix inequalities (LMI) and quadratic Lyapunov functions. The use of non-stationary Kalman filters is also discussed. A separation principle applies in some cases. A DC motor control setup shows the applicability of the approach in a real implementation.


*Keywords:* Time-varying sampling period; Unconventional sampling; Networked control; Kalman filter; LMI

## 1. Introduction

In computer control applications, the controller supposedly works at a fixed nominal period, but computer load, networks, sporadic faults, etc. may cause missing data, delays and sampling rate jitter, which can be significantly large, compromising performance or even stability. Controller design should take such situations into account, if possible. Several situations may arise regarding the information available to the controller about irregular sampling patterns: (1) *future* information: the controller code knows a priori the time interval during with the next control action will be applied; (2) *past* information: the controller code has access to a timer routine providing the time elapsed since last control action was applied; (3) no timing information available. By far, the second and third cases can be considered the most common in practice. For the second case, current processors can provide accurate measurements.

Networked control setups and alternative approaches to the one here considered also appear, for instance, in Albertos and Crespo (1999), Hu and Michel (2000), Tipsuwan and Chow (2003), Zhivoglyadov and Middleton (2003) and references therein. Missing data problems are studied in, for instance, Albertos, Sanchis, and Sala (1999).

This paper considers observer-based control under time-varying sampling period and (possibly time-varying as well) computation delay below one period. LMI quadratic decay-rate conditions will be set up to obtain suitable state-feedback and observer gains (allowing for time-varying gains if timing information is available, i.e., cases 1 and 2 above). Classical non-stationary Kalman filtering (Anderson & Moore, 1979) will be also discussed (in particular, it has been the choice for the experimental setup). Computation delay issues are also analysed by means of LMIs on an augmented model. For tractability, the LMIs must be set up for a finite number of sampling period and delay values, following a “gridding” approach (Apkarian & Adams, 1998). If grid points are close enough, performance constraints for intermediate points will be also fulfilled, but the approach needs a posteriori negative-definiteness checking.

The structure of the paper is as follows: next section will discuss preliminary definitions and notation.

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* Tel.: +34 96 387 9770. fax: +34 96 387 9579.
*E-mail address:* asala@isa.upv.es.

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The design methodology and separation principle appear in Section 3. Section 4 presents a DC motor experiment to illustrate the practical applicability of the results. Some conclusions are drawn in the last section.

## 2. Preliminaries

Let us have a $n$th-order linear time-invariant (LTI) continuous-time (CT) process:

$$\dot{x} = Ax(t) + Bu(t) \quad y(t) = Cx(t) \tag{1}$$

sampled at instants $t_k \in \mathbb{R}$, $k=0, 1, \ldots$, with $t_{k+1} > t_k$, $t_0 = 0$. The time-varying sampling period will be denoted by $T_k = t_{k+1} - t_k$, and it will be assumed to lie in a known compact set $\mathscr{T}$. Given the state at time $t_k$, $x(t_k)$, if input $u(t_k)$ is kept constant in the inter-sampling interval (i.e., a zero-order hold device, ZOH, is present), the state at time $t_{k+1}$ is

$$x(t_{k+1}) = \mathrm{e}^{AT_k} x(t_k) + \int_0^{T_k} \mathrm{e}^{A(T_k-\tau)} B \, \mathrm{d}\tau \, u(t_k) \tag{2}$$

or, introducing notation $A(T)=\mathrm{e}^{AT}$, $B(T)=\int_0^T \mathrm{e}^{A(T-\tau)} B \, \mathrm{d}\tau$, $A_k = A(T_k)$, $B_k = B(T_k)$, $x_k = x(t_k)$, $u_k = u(t_k)$:

$$x_{k+1} = A_k x_k + B_k u_k, \quad y_k = Cx_k, \tag{3}$$

i.e., system (1) appears as a linear time-varying discrete system. The regulator to be designed will be in the form

$$u_k = -F_k \hat{x}_k, \tag{4}$$

$$\hat{x}_{k+1} = (I - L_k C)(A_k \hat{x}_k + B_k u_k) + L_k y_{k+1}, \tag{5}$$

where $F_k$ and $L_k$ will be denoted as the controller and observer gains, respectively. These gains may depend on the time interval between the samples: in particular, at the moment when measurement of $y_k$ is made, a dependence on $T_{k-1}$ for the observer gain and $T_k$ for the controller one may be sought, if those periods are known.

It can be easily shown that, under this setting, current observer + state-feedback closed-loop equations can be expressed as (Antsaklis & Mitchel, 1997)

$$\phi_{k+1} = \begin{pmatrix} A_k - B_k F_k & B_k F_k \\ 0 & (I - L_k C) A_k \end{pmatrix} \phi_k, \tag{6}$$

where $\phi_k$ is a column vector, with dimension $2n$, formed by the plant state $x_k$ and the observer error $e_k = x_k - \hat{x}_k$.

Network and computation delays will be lumped into one value $\delta_k$ denoting the delay between sensing and actuation (on the following referred to as *input delay*), as depicted in Fig. 1. If the actuator replies with a time-stamp of the moment when a command is received, then $\delta_k$ will be known to the controller at $t_{k+1}$.

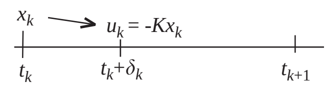


Fig. 1. Computation or network delay.

## 3. Controller design

As in the discrete constant-rate LTI case, the design of the output feedback controller (5) will be split into two subproblems: state-feedback and observer design. Separation principle issues will be discussed in Section 3.1.

A sufficient condition for stability with a decay rate $\mu$ for a LTI system $x_{k+1}=Gx_k$ is that there exists a positive definite matrix $P$ so that $G^{\mathrm{T}} P G - \mu^2 P < 0$ (Boyd, Gaoui, Feron, & Balakrishnan, 1994), so that $V(t)=x(t)^{\mathrm{T}} P x(t)$ is a Lyapunov function fulfilling $V(x_{k+1}) < \mu^2 V(x_k)$. If $G$ depends on a time-varying parameter $T$, as in (6), the inequality must be fulfilled for any possible $G(T)$ with a common $P$.

For stability analysis with time-varying sampling rates, a variation is proposed, defining $\mu=\mathrm{e}^{-\beta T}$, $\beta$ being a target CT equivalent exponential decay rate, i.e., looking for a feasible $P$ so that,

$$\Delta_\beta V(T) = G(T)^{\mathrm{T}} P G(T) - \mathrm{e}^{-2\beta T} P < 0 \quad \forall T \in \mathscr{T}. \tag{7}$$

The $\Delta_\beta$ notation has been introduced for later convenience. In this case, $V(t_k + T) < \mathrm{e}^{-2\beta T} V(t_k)$ for all $T \in \mathscr{T}$. Setting $\beta = 0$ amounts to checking only stability with no additional performance constraint. Decay rate conditions imply stability as, $\forall \beta > 0$, $\Delta_0 V(T) < \Delta_\beta V(T)$. Of course, as no assumptions about the statistical distribution of $T$ is made, the approach is conservative in some cases, and alternative approaches may be worth pursuing (Montestruque & Antsaklis, 2004).

From (6), in the state-feedback case $G(T) = A(T) - B(T)F(T)$; in the observer one, $G(T)=(I - L(T)C)A(T)$. A possible time-dependence in $F$ and $L$ has been introduced, to be later discussed.

Following (Boyd et al., 1994), it is easy to check that, if there exist feasible $X$ and $M(T)$ so that ($***$ denotes the necessary terms to build a symmetric matrix)

$$\begin{bmatrix} \mathrm{e}^{-2\beta T} X & *** \\ A(T)X - B(T)M(T) & X \end{bmatrix} > 0 \tag{8}$$

is verified for any $T \in \mathscr{T}$, the feedback gain $F(T) = M(T)X^{-1}$ stabilises $(A(T), B(T))$ with decay rate $\beta$, and $x_k^{\mathrm{T}} X^{-1} x_k$ is the associated Lyapunov function. (8) is obtained from (7) via the Schur complement formula. In the observer case, if $Q$ and $H(T)$ can be found so

$$\begin{bmatrix} \mathrm{e}^{-2\beta T} Q & *** \\ QA(T) - H(T)CA(T) & Q \end{bmatrix} > 0 \tag{9}$$