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Stability of Kalman filtering with Markovian packet losses☆

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Abstract

We consider Kalman filtering in a network with packet losses, and use a two state Markov chain to describe the normal operating condition of packet delivery and transmission failure. Based on the sojourn time of each visit to the failure or successful packet reception state, we analyze the behavior of the estimation error covariance matrix and introduce the notion of peak covariance, as an estimate of filtering deterioration caused by packet losses, which describes the upper envelope of the sequence of error covariance matrices $\{P_t, t\geqslant 1\}$ for the case of an unstable scalar model. We give sufficient conditions for the stability of the peak covariance process in the general vector case, and obtain a sufficient and necessary condition for the scalar case. Finally, the relationship between two different types of stability notions is discussed.

Keywords: Networked systems; Packet losses; Kalman filtering; Stability

1. Introduction

The problem of state estimation is of great importance in various applications ranging from tracking, detection and control, and in linear stochastic dynamical systems, Kalman filtering (Kailath, Sayed, & Hassibi, 2000; Kalman, 1960) plays an essential role. Recently there has been an increased research attention for filtering in distributed systems where sensor measurements and final signal processing take place in geographically separate locations and the usage of wireless or wireline communication channels is essential for data communication. In contrast to traditional filtering problems, an important feature in these networked systems is that the delivery of measurements to the estimator is not always reliable and losses of data may occur. This leads to estimation schemes which are required to handle missing data.

In this paper, we consider optimal filtering in a linear system with random packet losses. When the observer has full information about the loss of each packet, this leads to a modified filtering structure switching between the conventional Kalman filter when packets are received, and a deterministic predictor when a packet loss occurs.

We focus on the n dimensional linear time-invariant system

$$x_{t+1} = Ax_t + w_t, \quad t\geqslant 0,$$

where the initial state is x_0 at $t=0$. The sensor measurements are obtained starting from $t\geqslant 1$ in the form

$$y_t^0 = Cx_t + v_t, \quad t\geqslant 1,$$

where $C\in\mathbb{R}^{m\times n}$, and then y_t^0 is transmitted by a channel. Here $\{w_t, t\geqslant 0\}$ and $\{v_t, t\geqslant 1\}$ are two mutually independent sequences of independent and identically distributed (i.i.d.) Gaussian noises with covariance matrices Q and $R>0$, respectively. The two noise sequences are also independent of x_0, which is a Gaussian random vector with mean $\bar{x}_0 = Ex_0$ and covariance matrix P_{x_0}. The underlying probability space is denoted as $(\Omega, \mathbb{F}, \mathbb{P})$ where $\mathbb{F}$ is the σ-algebra of all events.

We consider a communication channel such that y_t^0 is exactly retrieved or the packet containing y_t^0 is lost due to corrupted data

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or substantial delay. When the packet is successfully received, one obtains the observation

$$y_t = y_t^0$$

and if there is a packet loss, by our convention, the observation obtained by the receiver is

$$y_t \equiv 0.$$

Under this assumption, the underlying communication link may be looked at as an erasure channel at the packet level.

We use $\gamma_t \in \{0, 1\}$ to indicate the arrival (with value 1) or loss (with value 0) of packets. Here γ_t may be interpreted as resulting from the physical operating condition of a network. Specifically, the state 0 for γ_t may correspond to channel error or network congestion which causes a straight packet loss or long delay resulting in packet dropping at the receiver. For facilitating the presentation, 0 and 1 shall be called the failure state and normal state, respectively. To capture the temporal correlation of the channel variation (e.g., in bursty error conditions), γ_t is modelled by a two state Markov chain with the transition matrix

$$\alpha = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}, \tag{1}$$

where p and q, respectively, are called the failure rate and recovery rate and $p, q > 0$. For instance, $1-p$ denotes the probability of the channel remaining at the normal state 1 after one step if it starts with state 1. This is usually called the Gilbert–Elliott channel model (Elliott, 1963; Gilbert, 1960). Obviously, a small value (close to 0) for p and a large value (close to 1) for q mean the channel is more reliable.

Based on the history $\mathbb{F}_t = \sigma(y_i, \gamma_i, i \leqslant t)$, which is the σ-algebra generated by the available information up to time t (i.e., all events that can be generated by these random variables), one can write a set of filtering and prediction equations corresponding to the optimal estimate $\hat{x}_t = E[x_t|\mathbb{F}_t]$ and $\hat{x}_{t+1|t} = E[x_{t+1}|\mathbb{F}_t]$, $t \geqslant 0$, respectively, by the same method as in Sinopoli et al. (2004) which dealt with the scenario of i.i.d. packet losses. We use the convention $\mathbb{F}_0 = \{\emptyset, \Omega\}$. The details for the recursion of $\hat{x}_t$ and $\hat{x}_{t+1|t}$ will not be repeated here. In this paper we focus on the estimation error of $\hat{x}_{t+1|t}$ with an associated prediction error covariance matrix

$$P_{t+1|t} \triangleq E(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'.$$

We write $P_{t+1|t} = P_{t+1}$. We use M' to denote the transpose of a vector or matrix M. To characterize the prediction error covariance, one can easily derive the following random Riccati equation

$$P_{t+1} = AP_tA' + Q - \gamma_t AP_tC'(CP_tC' + R)^{-1}CP_tA', \quad t \geqslant 1. \tag{2}$$

The initial condition in (2) is $P_1 = Var(x_1) = AP_{x_0}A' + Q$. Note that γ_t appears as a random coefficient in the recursion.

Under a Bernoulli i.i.d. packet loss modelling, the filtering stability may be effectively studied by a modified algebraic Riccati equation (MARE), which is obtained by replacing γ_t in Eq. (2) by the packet arrival rate λ. Subsequently, the analysis amounts to identifying a critical value λ_c such that stability holds if and only if the arrival rate is greater than λ_c (see Section 4 for additional discussion). This approach is generally termed as being based on the uncertainty threshold principle (Sinopoli et al., 2004). In contrast, when the channel model is given by a Markov chain, such a conversion into a deterministic MARE is no longer feasible, and since the channel is described by several independent parameters, the usual threshold argument is not applicable.

1.1. Background and related work

Filtering and estimation constitute an important aspect in sensor network deployment for monitoring, detection or tracking (Chong & Kumar, 2003; Zhang, Moura, & Krogh, 2005; Zhao, Shin, & Reich, 2002), as well as multi-vehicle coordination (Varaiya, 1993), since in reality sensors can only obtain noisy information about a physical activity in its vicinity. And for many linear stochastic models, a useful tool is the standard Kalman filtering theory which has been widely used in various estimation and control scenarios. Recently there is an increased attention for its application in distributed networks while new theoretical questions and implementation issues emerge. In close relation to estimation in lossy sensor networks, there also has been a long history of research on filtering with missing signals at certain points of time, i.e., the output does not necessarily contain the signal in question and it may be only a noise component. Such models were referred to as systems with uncertain observations; see (Hadidi & Schwartz, 1979; Jaffer & Gupta, 1971; Nahi, 1969; Tugnait, 1981). The early work (Nahi, 1969) considered optimal state estimation within the class of linear filters; by modelling the uncertainty via a sequence of i.i.d. binary random variables indicating the signal availability, the author derived a recursion similar to the Kalman filter utilizing the statistics of the unobserved binary uncertainty sequence (Nahi, 1969). The work (Hadidi & Schwartz, 1979) gave conditions for obtaining recursive filtering when the uncertainty sequence is not necessarily i.i.d. Asymptotic stability of the MMSE filter was established in Tugnait (1981) when the loss sequence is i.i.d. with known loss probability; since in this case the estimation covariance is governed by a deterministic equation, one can obtain stability analysis by constructing an equivalent linear system without data losses.

In the more recent research on network models, (Fletcher, Rangan, & Goyal, 2004; Smith & Seiler, 2003) considered state estimation with lossy measurements resulting from time-varying channel conditions. In particular, Smith and Seiler (2003) developed a suboptimal jump linear estimator for complexity reduction in computing the corrector gain using finite loss history where the loss process is modelled by a two state Markov chain. The work (Fletcher et al., 2004) introduced a more general multiple state Markov chain to model the loss and nonloss channel states, and the asymptotic mean square estimation error for suboptimal linear estimators is analyzed and op-