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Brief paper

Frequency domain maximum likelihood estimation of linear dynamic errors-in-variables models☆

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Abstract

This paper studies the linear dynamic errors-in-variables problem for filtered white noise excitations. First, a frequency domain Gaussian maximum likelihood (ML) estimator is constructed that can handle discrete-time as well as continuous-time models on (a) part(s) of the unit circle or imaginary axis. Next, the ML estimates are calculated via a computationally simple and numerically stable Gauss–Newton minimization scheme. Finally, the Cramér–Rao lower bound is derived.

Keywords: Errors-in-variables; Maximum likelihood; Continuous-time; Discrete-time; Frequency domain

1. Introduction

Linear dynamic errors-in-variables (EIV) modelling is important in those applications where one is looking for a better understanding of the underlying input–output relation of a process rather than making output predictions from noisy observations. One can distinguish between two cases: either the excitation of the process can be freely chosen, or one has to live with the operational (natural) perturbations. If the excitation can be freely chosen, then it is strongly recommended to use periodic excitation signals because it significantly simplifies the identification problem: (i) non-parametric estimates of the disturbing noise (co-)variances are obtained in a preprocessing step, and (ii) since mutually correlated, coloured input/output errors are allowed, identification in feedback is just a special case of the general framework (see Pintelon & Schoukens, 2001). In the second case the excitation is often random and parts of it may even be unmeasurable. This paper handles the second case, assuming that the excitation is a stochastic process with rational power spectrum. As will be shown in the sequel of the paper, the second case is much more complicated than the first: besides the plant model one should also identify simultaneously the signal model, and the input/output noise models.

Identifiability is a first key issue in EIV modelling: under which conditions on the excitation, the input/output errors, and the process is the EIV problem uniquely solvable? This question has been studied in detail in econometrics and an extensive literature is available (see Söderström, 2006a for an exhaustive overview). For example, Anderson and Deistler (1984) handles the identifiability of scalar EIV problems with coloured input/output errors, while Nowak (1993) covers the multivariable case. The results of Anderson and Deistler (1984) have been generalized in Castaldi and Soverini (1996) and Agüero and Goodwin (2006).

A second key issue is the numerical calculation of the EIV estimates. Several algorithms have been proposed, each of them having their specific advantages and disadvantages (see Söderström, 2006a for an exhaustive overview). For example, the statistically efficient time domain maximum likelihood (ML) method has high computational complexity, while the computationally simpler instrumental variable methods have low statistical accuracy (Söderström, 2006a). This paper presents the frequency domain version of the time domain Gaussian ML estimator (Söderström & Stoica, 1989). Besides the slightly different handling of the transient effects (in the frequency domain the transient depends on the initial AND the final conditions), the spectral factorization is also carried

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out differently in the frequency domain (no Riccati equation must be solved in each iteration step). The advantages of the frequency domain approach are that it is equally simple to identify continuous-time (CT) models as discrete-time (DT) ones, that filtering of the input/output signals reduces to the selection of the appropriate frequencies in the input/output spectra, that time domain signals as well as frequency domain spectra can be handled, and that improper systems (order numerator > order denominator) can be identified. The latter is important in, for example, the modelling of electrical machines (Kamwa, Viarouge, Le-Huy, & Dickinson, 1992).

A third issue is the identification of CT models. Except for Mahata and Garnier (2005) and Söderström, Larsson, Mahata, and Mossberg (2006), all methods handle the DT case and no algorithms for direct CT EIV modelling are available. The time domain methods in Mahata and Garnier (2005) and Söderström et al. (2006) identify CT models in the presence of white input/output errors, and Mahata and Garnier (2005) also handle the nonuniformly sampled data case. The approach presented in this paper identifies DT as well as CT models from measured frequency domain spectra or uniformly sampled time domain signals disturbed by coloured input/output errors.

A fourth issue is filtering of the measured input/output signals: often one is only interested in the plant characteristics on a part of the unit circle (or imaginary axis); or one would like to remove the effect of trends (low-frequency range), disturbances (mains, high-frequency noise, ...), and errors that cannot be written as filtered white noise (e.g. sinewave with time-varying frequency); or for reasons of generating easily high-quality starting values, a high (infinite)-dimensional system is approximated in each frequency band by a low-order model. The prefiltering does not affect the input/output relationship, and is equivalent to dividing the input/output noise models by the prefilter characteristics. To preserve the efficiency and/or consistency of the identified plant model, the input/output noise models should be flexible enough to follow the input/output error spectra accurately and, as such, they will try to cancel the effect of the prefilter. Hence, through the prefilter and input/output noise model selection a compromise must be made between suppression of the undesired frequency bands and the loss in consistency and/or efficiency of the plant estimates (see Ljung, 1999 for the generalized output error case). These conflicting demands which are inherent to all time domain methods are avoided by the frequency domain approach presented in this paper: the plant, signal, and input/output noise models are identified in the frequency band(s) of interest only. Note that if the model structure (plant, signal, and noise models) is valid in a larger frequency band than the one used for identification, then some information is lost through the filtering, and the variability of the estimates will increase. This is valid for both the time and frequency domain approaches.

Summarized the contributions of this paper are:

1. a (computationally simple) frequency domain Gaussian ML estimator is developed for the general case of coloured and mutually independent input/output errors,
2. the ML estimator can handle DT as well as CT modelling on (a) part(s) of the unit circle or imaginary axis,
3. a numerically stable Gauss–Newton minimization scheme of the ML cost function is derived,
4. easy to implement and numerical stable calculation of the Cramér–Rao (CR) lower bound.

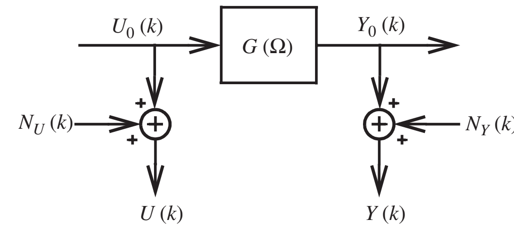

Fig. 1. Open-loop errors-in-variables set up where $\Omega = z^{-1}$ for discrete-time systems and $\Omega = s$ for continuous-time systems.

2. Open-loop EIV identification

2.1. EIV stochastic framework

Consider the open-loop setup of Fig. 1

$$Y(k) = G(\Omega_k)U_0(k) + N_Y(k),$$

$$U(k) = U_0(k) + N_U(k) \tag{1}$$

with $U_0(k)$ and $Y_0(k) = G(\Omega_k)U_0(k)$ the true unknown input and output spectra; $G(\Omega)$ the plant transfer function; $N_U(k)$ and $N_Y(k)$ the input and output measurement errors; $\Omega_k = z_k^{-1} = \exp(-\mathrm{j}2\pi f_k/f_s)$, with f_s the sampling frequency, for DT systems; $\Omega_k = s_k = \mathrm{j}2\pi f_k$ for CT systems; and k the frequency index. The input measurement noise $N_U(k)$, the output measurement noise $N_Y(k)$, and the excitation $U_0(k)$ are modelled as ARMA stochastic processes

$$U_0(k) = L(\Omega_k)E_L(k),$$

$$N_U(k) = H_U(\Omega_k)E_U(k),$$

$$N_Y(k) = H_Y(\Omega_k)E_Y(k), \tag{2}$$

where $L(\Omega)$, $H_U(\Omega)$, and $H_Y(\Omega)$ are the signal, the input noise, and the output noise transfer functions, respectively; and where $E_L(k)$, $E_U(k)$, and $E_Y(k)$ are the signal, the input, and the output driving white noise sources, respectively.

Assumption 1 (*class of EIV systems*).

(1) the input/output data are generated by (1) and (2),
(2) $G(\Omega)$, $L(\Omega)$, $H_U(\Omega)$, and $H_Y(\Omega)$ are rational functions of Ω with real coefficients,
(3) $E_L(k)$, $E_U(k)$, and $E_Y(k)$ are independent (mutually, and over the frequency index k), circular complex ($E\{E^2(k)\}=0$, with $E\{\}$ the expected value, and $E = E_L$, E_U, and E_Y) normally distributed noise, with zero mean ($E\{E(k)\}=0$), and variances λ_L, λ_U, and λ_Y, respectively,