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Brief paper

Reliable mixed $\mathscr{L}_2/\mathscr{H}_\infty$ fuzzy static output feedback control for nonlinear systems with sensor faults[☆]

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Abstract

This paper is concerned with the design of reliable mixed $\mathscr{L}_2/\mathscr{H}_\infty$ static output feedback (SOF) fuzzy controllers for nonlinear continuous-time systems with complete sensor faults. The Takagi and Sugeno (T–S) fuzzy model is employed to represent a nonlinear system. A sufficient condition for the existence of reliable mixed $\mathscr{L}_2/\mathscr{H}_\infty$ SOF fuzzy controllers is presented in terms of a set of quadratic matrix inequalities (QMIs), which not only guarantees that the closed-loop fuzzy system satisfies a desired $\mathscr{H}_\infty$ disturbance attenuation constraint for all admissible operating regimes (including the normal and sensor fault cases), but also provides different upper bounds on the $\mathscr{L}_2$ performance criterion for different operating regimes. A suboptimal reliable fuzzy controller is obtained by the proposed iterative linear matrix inequality (ILMI) algorithm for minimizing the normal $\mathscr{L}_2$ performance bound, while maintaining acceptable lower levels of the bounds in the sensor fault cases. Finally, a numerical example is given to illustrate the effectiveness of the proposed design method.

Keywords: Fuzzy control; $\mathscr{H}_\infty$ control; Nonlinear systems; Optimal control; Reliable control; Sensor faults

1. Introduction

In recent years, the study of reliable control has received considerable attention due to the growing demands on reliability. The main task of this study is to design a fixed controller such that the closed-loop system can maintain stability and performance, not only when all control components are operational, but also in case of some admissible control component outages. Reliable control problems for linear systems have been extensively studied, and several approaches have been proposed, such as coprime factorization approach (Vidyasagar & Viswanadham, 1985), algebraic Riccati equation (ARE) approach (Veillette, 1995; Veillette, Medanic, & Perkins, 1992; Yang, Wang, & Soh, 2000), pole region assignment technique (Zhao & Jiang, 1998), linear matrix inequality (LMI) approach (Liao, Wang, & Yang, 2002; Yang, Yee, & Wang, 2002). Recently, the reliable control problems for nonlinear systems have been developed by using the Hamilton–Jacobi inequality (HJI) approach. For example, Yang, Lam, and Wang (1998) extended the results of Veillette et al. (1992) to the study of reliable $\mathscr{H}_\infty$ control problem for nonlinear systems with sensor and/or actuator faults; Liang, Liaw, and Lee (2000) studied the reliable linear quadratic (LQ) regulator problem for nonlinear systems with actuator faults, which extended the results of Veillette (1995). However, different from their counterparts for linear systems, it is very difficult to solve HJIs either analytically or numerically.

Recent years have witnessed rapidly growing interest in fuzzy control of nonlinear systems. The numerous successful applications have sparked a flurry of activities in the study of fuzzy control. Especially, the control technique

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based on the so-called Takagi–Sugeno (T–S) fuzzy model (Takagi & Sugeno, 1985) has attracted lots of attention (Cao, Rees, & Feng, 1996; Chen, Tseng, & Uang, 2000; Nguang & Shi, 2003; Tanaka & Wang, 2001; Teixeira, Assunção, & Avellar, 2003; Tuan, Apkarian, Narikiyo, & Yamamoto, 2001 and references therein), since it is conceptually simple and straightforward. First, the nonlinear system is represented or approximated by a T–S fuzzy model. In this type of fuzzy model, local dynamics in different state space regions are represented by linear models. The overall model of the system is obtained by "blending" of these linear models through fuzzy membership functions. Then, based on this fuzzy model, a model-based fuzzy control is developed to achieve the stability and performance for nonlinear systems. This fuzzy modeling approach provides a powerful tool for modeling complex nonlinear systems. More recently, based on the T–S fuzzy model, the reliable LQ state feedback fuzzy control problem for nonlinear systems with actuator faults has been proposed in Wu (2004a,b).

When controlling a real plant, it is desirable to design a control system with guaranteed $\mathscr{L}_2$(or $\mathscr{H}_2$) performance under a prescribed $\mathscr{H}_\infty$ disturbance attenuation constraint (see Bernstein & Haddad, 1989; Chen et al., 2000; Doyle, Zhou, Glover, & Bodenheimer, 1994; Khargonekar & Rotea, 1991; Limebeer, Anderson, & Hendel, 1994; Lin, 1996). Recently, the reliable mixed $\mathscr{L}_2$(or $\mathscr{H}_2$)/$\mathscr{H}_\infty$ control problem for linear systems has been proposed in Yang et al. (2002) and Liao et al. (2002). However, to the best of our knowledge, no result on reliable mixed $\mathscr{L}_2/\mathscr{H}_\infty$ control for nonlinear systems is available. Furthermore, it is not always possible to have access to all of the state variables in nonlinear systems, and the dynamic output feedback results in high order controllers which may not be practical in industry. In this situation, the static output feedback (SOF) control is more suitable for practical application.

In this study, a reliable mixed $\mathscr{L}_2/\mathscr{H}_\infty$ SOF fuzzy control design for nonlinear systems with complete sensor faults is proposed to achieve a suboptimal $\mathscr{L}_2$ performance with a desired $\mathscr{H}_\infty$ disturbance attenuation constraint. The T–S fuzzy model is employed to represent a nonlinear system. A sufficient condition for the existence of reliable mixed $\mathscr{L}_2/\mathscr{H}_\infty$ SOF fuzzy controllers is presented in terms of a set of quadratic matrix inequalities (QMIs), which not only guarantees that the closed-loop fuzzy system satisfies a desired $\mathscr{H}_\infty$ constraint for all admissible operating regimes (including the normal and sensor fault cases), but also provides different upper bounds on the $\mathscr{L}_2$ performance criterion for different operating regimes. Since different Lyapunov matrices are adopted for different operating regimes, it leads to a less conservative controller design than by using a common Lyapunov matrix. Moreover, its other advantage is that a suboptimal reliable fuzzy controller can be designed to optimize the normal $\mathscr{L}_2$ performance bound while maintaining acceptable lower levels of the bounds in the sensor fault cases, because it is often undesirable to sacrifice significantly the normal $\mathscr{L}_2$ performance for the occasional fault cases. Furthermore, by using LMI optimization techniques (Boyd, Ghaoui, Feron, & Balakrishnan, 1994; Gahinet, Nemirovski, Laub, & Chilali, 1995), the addressed suboptimal reliable fuzzy control problem can be treated by the proposed iterative LMI (ILMI) approach.

The paper is organized as follows. The problem formulation is presented in Section 2. In Section 3, a sufficient condition for the existence of reliable mixed $\mathscr{L}_2/\mathscr{H}_\infty$ SOF fuzzy controllers is provided. In Section 4, an ILMI algorithm for the suboptimal reliable fuzzy controller is proposed. In Section 5, a numerical example is provided to demonstrate the effectiveness of the proposed method. Finally, concluding remarks are made in Section 6.

Notations: The superscript "T" denotes matrix transpose. The symbol $*$ is used to denote the transposed elements in the symmetric positions of a matrix. The notation $\Theta(h)$ stands for $\sum_{i=1}^{r} h_i(\boldsymbol{\theta}(t))\Theta_i$ with $\Theta \in \{\boldsymbol{A},\ \boldsymbol{B}_1,\ \boldsymbol{B}_2, \boldsymbol{C}_2,\ \boldsymbol{K}\}$. $\mathrm{E}\{\cdot\}$ denotes the expectation operator.

2. Problem formulation

Consider a continuous-time nonlinear system which can be described by the following T–S fuzzy model:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\theta}(t))\{\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_{1i}\boldsymbol{w}(t) + \boldsymbol{B}_{2i}\boldsymbol{u}(t)\}, \tag{1}$$

$$\boldsymbol{z}(t) = \boldsymbol{C}_1\boldsymbol{x}(t) + \boldsymbol{D}_1\boldsymbol{u}(t), \tag{2}$$

$$\boldsymbol{y}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\theta}(t))\boldsymbol{C}_{2i}\boldsymbol{x}(t), \tag{3}$$

where $\boldsymbol{\theta}(t) = [\theta_1(t),\ \theta_2(t), \ldots, \theta_g(t)]^{\mathrm{T}}$ is the premise variable vector, r is the number of model rules. $h_i(\boldsymbol{\theta}(t))$ is the normalized weight for each rule, that is, $h_i(\boldsymbol{\theta}(t)) \geqslant 0$ and $\sum_{i=1}^{r} h_i(\boldsymbol{\theta}(t)) = 1$ for all t. $\boldsymbol{x}(t) \in \Re^n$ is the state, $\boldsymbol{u}(t) \in \Re^m$ is the control input, $\boldsymbol{w}(t) \in \Re^d$ is the exogenous disturbance which belongs to $\mathscr{L}_2[0, \infty)$, $\boldsymbol{z}(t) \in \Re^p$ is the controlled output, $\boldsymbol{y}(t) \in \Re^q$ is the measurement output, $\boldsymbol{A}_i$, $\boldsymbol{B}_{1i}$, $\boldsymbol{B}_{2i}$, $\boldsymbol{C}_1$, $\boldsymbol{D}_1$, and $\boldsymbol{C}_{2i}$ are known constant matrices with appropriate dimensions. In this paper, we assume that $\boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{C}_1 = 0$ and $\boldsymbol{R} = \boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{D}_1 > 0$.

We consider the following SOF fuzzy controller for the fuzzy system (1)–(3):

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\theta}(t))\boldsymbol{K}_i\boldsymbol{y}(t), \tag{4}$$

where $\boldsymbol{K}_i \in \Re^{m \times q}$, $i \in \mathscr{S}_r \triangleq \{1, 2, \ldots, r\}$ are feedback gain matrices to be determined.

Definition 1 (*Sensor failure*). The sth sensor is said to have failed at time $T_f > 0$ if $y_s(t) = 0\ \forall t > T_f$.

Let $\boldsymbol{y}^{\mathrm{F}}(t)$ represent the measurement output after failures have occurred. Then the following sensor fault model is