PERGAMON

Automatica 37 (2001) 2011–2016

automatica

www.elsevier.com/locate/automatica

Brief Paper

On the design of ILC algorithms using optimization☆

Svante Gunnarsson*, Mikael Norrlöf

Department of Electrical Engineering, Linköping University, SE-58183 Linköping, Sweden

Received 24 November 1999; revised 22 August 2000; received in final form 31 May 2001

Abstract

Iterative learning control (ILC) based on minimization of a quadratic criterion in the control error and the input signal is considered. The focus is on the frequency domain properties of the algorithm, and how it is able to handle non-minimum phase systems. Experiments carried out on a commercial industrial robot are also presented. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Learning control; Optimization; Robotics; Frequency domains; Non-minimum phase systems

1. Introduction

The purpose of this paper is to show some new aspects of an ILC algorithm derived using optimization. Parts of the results presented here can also be found in Gunnarsson and Norrlöf (1999). The general setup is of standard ILC type, i.e. the system to be controlled is carrying out the same operation repeatedly, the desired operation is carried out during a finite time interval and the purpose of ILC is to obtain good servo properties. General introductions to the area of ILC are given in e.g. Moore (1993, 1998), and a collection of recent results is found in Bien and Xu (1998). More specifically, this paper deals with ILC applied to linear SISO systems working in discrete time. The general system description will then be

$$y_k(t) = T_r(q)r(t) + T_u(q)u_k(t), \tag{1}$$

where $u_k(t)$ and $y_k(t)$ denote the ILC input signal and the output signal, respectively. Furthermore, $r(t)$ denotes the desired output (reference) signal. The subscript k denotes iteration number. The reference signal is the same in all iterations while the other signals will change from iteration to iteration. All signals are defined on a finite time interval $t = 0, \ldots, N$. Finally, $T_r(q)$ and $T_u(q)$ are stable discrete time filters, where q denotes the shift operator.

The formulation in Eq. (1) is taken from Norrlöf (2000), and it covers a wide class of situations ranging from an open loop control problem to a closed loop system operating under both feed-back and feed-forward control as depicted in Fig. 1, where the signal $u_k(t)$ represents a signal added to the signals normally generated in the control system. ILC is used as a complement to the conventional robot control system. A slight modification of the system structure shown in Fig. 1 is to let the ILC input signal be used as a feed-forward signal to the control signal generated by the feed-back and feed-forward parts of the controller. This just corresponds to a redefinition of the transfer operator $T_u(q)$ in Eq. (1).

The fundamental problem in ILC is to design an update algorithm for the input signal $u_k(t)$ such that the error $e_k(t) = r(t) - y_k(t)$ is minimized in some appropriate sense. A general updating equation is given by

$$u_{k+1}(t) = Q(q)(u_k(t) + L(q)e_k(t)), \tag{2}$$

where $Q(q)$ and $L(q)$ are linear, possibly non-causal, filters. The choice of $Q(q)$ and $L(q)$ is the main issue in the design of an ILC algorithm. The aim in this paper is to show that an ILC algorithm derived using optimization can be seen as a particular choice of the filters $L(q)$ and $Q(q)$. The paper is organized as follows. In Section 2, an ILC algorithm is derived using optimization. Then in Section 3, it is shown how this type of ILC algorithm can deal with non-minimum phase systems. In Section 4, the frequency domain properties of the algorithm are investigated and in Section 5, the ILC algorithm is applied to an

☆This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Li-Xin Wang under the direction of Editor Frank L. Lewis. This work was supported by CENIIT at Linköping University and by ABB Robotics within ISIS at Linköping University.

* Corresponding author. Tel.: + 46-13-281747; fax: + 46-13-282622.
E-mail address: svante@isy.liu.se (S. Gunnarsson).

PII: S0005-1098(01)00154-6

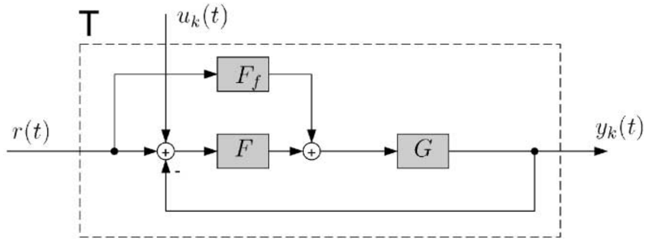

Fig. 1. An example of a realization of the system in Eq. (1).

industrial robot with good results. Finally, some conclusions are given in Section 6.

2. ILC using optimization

The optimization approach to ILC is well known, and previous contributions can be found in e.g. Gorinevsky, Torfs, and Goldenberg (1995), Frueh and Phan (1998), Lee, Lee, and Kim (2000), and Amann, Owens, and Rogers (1995). The aim of this paper is to investigate some specific aspects of this approach, namely the interpretation of the algorithm in the frequency domain, the ability to handle non-minimum phase systems, and the use in experiments on a real robot. Introduce the vectors

$$\mathbf{Y}_k = (y_k(0), \ldots, y_k(N))^{\mathrm{T}}, \tag{3}$$

$$\mathbf{U}_k = (u_k(0), \ldots, u_k(N))^{\mathrm{T}}, \tag{4}$$

$$\mathbf{R} = (r(0), \ldots, r(N))^{\mathrm{T}} \tag{5}$$

and $\mathbf{E}_k = \mathbf{R} - \mathbf{Y}_k$, where k denotes the iteration number and $t = 0, \ldots, N$ denote the sampling points. Using these notations the system can be described by the equation

$$\mathbf{Y}_k = \mathbf{T}_r\mathbf{R} + \mathbf{T}_u\mathbf{U}_k, \tag{6}$$

where $\mathbf{T}_u$ is a matrix formed by the impulse response coefficients of the transfer operator $T_u(q)$, i.e.

$$\mathbf{T}_u = \begin{pmatrix} t_0 & 0 & 0 & \ldots & 0 \\ t_1 & t_0 & 0 & \ldots & 0 \\ \ddots & \ddots & \ddots & \ddots & \\ t_N & t_{N-1} & \ldots & t_1 & t_0 \end{pmatrix}, \tag{7}$$

and $\mathbf{T}_r$ is defined analogously. In (6), it is also assumed that the initial conditions are zero, i.e. that $y_k(t) = 0$ for $t < 0$. In Eq. (7) the matrix elements along the diagonals are constant but this is not necessary. In case the system is time varying in the sense that the dynamics change during one iteration it is straightforward to let the coefficients in $\mathbf{T}_u$ vary along the diagonals. It is however, assumed that the same $\mathbf{T}_u$ is valid in each iteration, i.e. slow changes in the system to be controlled are not covered. For completeness the diagonal element t_0 is included, but the discussion is not restricted to this case. The discussion here is restricted to SISO systems. Optimization based LIC in the MIMO case is dealt with in e.g. Lee et al. (2000). The matrices in Eq. (6) will depend on the structure of the control system. The work presented here represents a situation when the system is controlled using feed-forward and feed-back according to Fig. 1. The matrices in Eq. (6) are then given by

$$\mathbf{T}_r = (\mathbf{I} + \mathbf{GF})^{-1}\mathbf{G}(\mathbf{F}_{\mathrm{f}} + \mathbf{F}), \qquad \mathbf{T}_u = (\mathbf{I} + \mathbf{GF})^{-1}\mathbf{GF}, \tag{8}$$

where $\mathbf{F}$ and $\mathbf{F}_{\mathrm{f}}$ are the matrices corresponding to the feed-back and feed-forward transfer operators.

The idea is now to determine $\mathbf{U}_{k+1}$ such that the error $\mathbf{E}_{k+1}$ becomes as small as possible by minimizing the criterion

$$\mathbf{J} = \mathbf{E}_{k+1}^{\mathrm{T}}\mathbf{W}_e\mathbf{E}_{k+1} + \mathbf{U}_{k+1}^{\mathrm{T}}\mathbf{W}_u\mathbf{U}_{k+1}, \tag{9}$$

where $\mathbf{W}_e$ and $\mathbf{W}_u$ are positive definite weight matrices determining the trade off between performance and input energy. The weight matrices can be used for both time and frequency weighting but this possibility will not be exploited here. An example of the use of non-diagonal weighting matrices is presented in Chapter 15 of Norrlöf (2000). The criterion is minimized, subject to the constraint

$$(\mathbf{U}_{k+1} - \mathbf{U}_k)^{\mathrm{T}}(\mathbf{U}_{k+1} - \mathbf{U}_k) \leqslant \delta. \tag{10}$$

Introduction of a Lagrange multiplier, differentiation, and use of the expression

$$\mathbf{E}_{k+1} = (\mathbf{I} - \mathbf{T}_r)\mathbf{R} - \mathbf{T}_u\mathbf{U}_{k+1}, \tag{11}$$

where $\mathbf{T}_u$ denotes a nominal model, gives

$$\mathbf{U}_{k+1} = (\mathbf{W}_u + \lambda\cdot\mathbf{I} + \mathbf{T}_u^{\mathrm{T}}\mathbf{W}_e\mathbf{T}_u)^{-1} (\lambda\mathbf{U}_k + \mathbf{T}_u^{\mathrm{T}}\mathbf{W}_e(\mathbf{I} - \mathbf{T}_r)\mathbf{R}), \tag{12}$$

which can be reformulated into

$$\mathbf{U}_{k+1} = \mathbf{Q}(\mathbf{U}_k + \mathbf{L}\mathbf{E}_k), \tag{13}$$

where

$$\mathbf{Q} = (\mathbf{W}_u + \lambda\cdot\mathbf{I} + \mathbf{T}_u^{\mathrm{T}}\mathbf{W}_e\mathbf{T}_u)^{-1}(\lambda\cdot\mathbf{I} + \mathbf{T}_u^{\mathrm{T}}\mathbf{W}_e\mathbf{T}_u), \tag{14}$$

and

$$\mathbf{L} = (\lambda\cdot\mathbf{I} + \mathbf{T}_u^{\mathrm{T}}\mathbf{W}_e\mathbf{T}_u)^{-1}\mathbf{T}_u^{\mathrm{T}}\mathbf{W}_e. \tag{15}$$

The updating matrices $\mathbf{Q}$ and $\mathbf{L}$ hence depend on the nominal model $\mathbf{T}_u$ and the weight matrices $\mathbf{W}_u$ and $\mathbf{W}_e$. The Lagrange multiplier λ is not computed explicitly but instead used as a design variable. By putting $\mathbf{W}_u = 0$, i.e.