Control Engineering Practice 55 (2016) 69–79

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Nonlinear robust observer design using an invariant manifold approach

Irfan Ullah Khan *, David Wagg, Neil D. Sims

Department of Mechanical Engineering, The University of Sheffield, Sheffield, UK

ARTICLE INFO

Article history:
Received 10 February 2016
Received in revised form
23 June 2016
Accepted 23 June 2016
Available online 15 July 2016

Keywords:
Observer design
Invariant manifold
Lipschitz non-linearity
Error dynamics
Mapping functions

ABSTRACT

This paper presents a method to design a reduced order observer using an invariant manifold approach. The main advantages of this method are that it enables a systematic design approach, and (unlike most nonlinear observer design methods), it can be generalized over a larger class of nonlinear systems. The method uses specific mapping functions in a way that minimizes the error dynamics close to zero. Another important aspect is the robustness property which is due to the manifold attractivity: an important feature when an observer is used in a closed loop control system. A two degree-of-freedom system is used as an example. The observer design is validated using numerical simulation. Then experimental validation is carried out using hardware-in-the-loop testing. The proposed observer is then compared with a very well known nonlinear observer based on the off-line solution of the Riccati equation for systems with Lipschitz type nonlinearity. In all cases, the performance of the proposed observer is shown to be very high.

1. Introduction

For nonlinear systems the theory of linear observer design has been extended e.g. extended Luenburger observer (Price & Cook, 1982; Zeitz, 1987) or extended Kalman filter (Boutayeb & Aubry, 1999; Mercorelli, 2012). As a result estimation is limited to a small domain and requires high computation power. Thau (1973) and then Kou, Elliott, and Tarn (1975) were the first to attempt nonlinear techniques for the observer design. Since then a lot of work has been done on the observer design using nonlinear theory but mostly limited to certain classes of system that cannot typically be generalized to other classes of systems.

The observers based on Lyapunov theory give sufficient conditions for the existence of the observer for nonlinear systems (Chaoui, Golbon, Hmouz, Souissi, & Tahar, 2015; Nikoobin & Haghighi, 2009; Vaclavek & Blaha, 2005). It may be possible for the low order nonlinear systems to satisfy the conditions presented in the theorems based on Lyapunov theory but it is very difficult to find higher order nonlinear systems that can satisfy those conditions (Lageman, Mahony, & Trumpf, 2008). The observers based on extended linearization techniques linearize the error dynamics through a nonlinear output injection function (Huang, Xu, Han, & Lam, 2001; Li, Yang, Chen, & Chen, 2012; Talole, Kolhe, & Phadke, 2010). This type of observer functions locally at a fixed point and for multi-input multi-output systems the design methodology can be very complicated.

For nonlinear observers, designs based on Lie-algebraic theory have also been used in the literature (Lageman et al., 2008, 2010; Maithripala et al., 2004). In these techniques, the problems linked with nonlinear observer design have been dealt with by using linear techniques that exploit linear observer theory. One of the advantages of using Lie-algebraic theory over the extended linearization techniques is that in the former case the observer is valid in any region where the transformation exists, whereas in the latter case the observer is designed at a fixed point. This method can also be used to design observer for multi-input multi-output systems. The down side of this technique is that the nonlinear system must satisfy a non-generic condition along with the finding of a necessary state transformation, which is not an easy task.

Generally there are two ways to deal with observer design in nonlinear systems (Astolfi, Karagiannis, & Ortega, 2007). If the system nonlinearities are a linear function of unmeasured states or are monotonic, then observers based on linear theory can be used, or passivity can be exploited. Alternatively, the observer requires the existence of an attractive and invariant manifold. These types of observers comprise a linear filter and nonlinear output mapping

* Corresponding author.
E-mail addresses: iukhan1@sheffield.ac.uk (I.U. Khan), david.wagg@sheffield.ac.uk (D. Wagg), n.sims@sheffield.ac.uk (N.D. Sims).

http://dx.doi.org/10.1016/j.conengprac.2016.06.015
0967-0661/

functions. The theory of sliding mode has also been used to design observers for both linear and nonlinear systems (Edwards & Tan, 2006; Nollet, Floquet, & Perruquetti, 2008).

The observer design in the sliding mode methodology resembles the one proposed in this paper up to the extent of defining an asymptotically stable surface. In the sliding mode observer, the sliding surface is defined in terms of the error between the estimated and known states and a discontinuous/switching function is defined to bring the error dynamics to the sliding surface (Edwards, Spurgeon, Tan, & Patel, 2007; Perruquetti & Barbot, 2002; Slotine, Hedrick, & Misawa, 1987), whereas in the proposed approach the observer design is reduced to make the error dynamics asymptotically stable, which depends on the definition of some mapping functions. The sliding mode observer is known for its insensitivity to parameter variation and disturbance rejection but the observer matching condition restricts the applicability of the sliding mode observer and the system has to be minimum phase (Fridman, Shtessel, Edwards, & Yan, 2008; Kalsi, Lian, Hui, & Zak, 2009). This means that all the zeros of the system should be on the left hand side, or in other words the internal dynamics of the system need to be stable for the design of first order sliding mode observer. To overcome this issue higher order sliding mode observers are proposed (Floquet & Barbot, 2006; Fridman, Levant, & Davila, 2007). However, the technique proposed in the present contribution could be extended, in a similar fashion for non-minimum phase systems (Astolfi et al., 2007).

Mainly there are two types of sliding mode observers. The type based on equivalent control methods are Utkin observers and the type based on Lyapunov methods are Walcott and Zak observers (Walcott & Zak, 1986). The Utkin sliding mode observer (Drakunov & Utkin, 1995) does not have a static observer gain. The disadvantage of not having a static observer gain is that the state estimation can be performed only with the bounded error and not asymptotically. The Walcott and Zak observer has a static observer gain and the error is reduced based on system uncertainty. Another disadvantage of traditional sliding mode observers is high frequency switching action.

In Zhu and Han (2002), Zhang, Su, Wang, and Han (2012), Zemouche, Boutayeb, and Bara (2008), and Ha and Trinh (2004) the observer designs based on the solution of the Riccati equation are proposed for systems with Lipschitz type nonlinearity. In all these papers, to check the validity of the observer, the only test performed is that different initial conditions are given to the actual system and it is shown that the observer is converging. There is no discussion about the robustness of the observers against parameter variation, measurement noise or external disturbance. In this paper in addition to the initial condition test, both the proposed observer and the observer based on the off-line solution of the Riccati equation are tested for robustness against parameter variation, measurement noise and external disturbance.

The theory for observer developed by Astolfi et al. (2007) has been implemented on many systems, such as ball and beam system, range estimation in a vision system and magnetic levitation system. The present contribution builds upon these previous studies by demonstrating application of the observer to a real mechanical system both in open loop and closed loop, so that the robustness to parameter variation, external disturbance and measurement noise can be explored for the first time. Therefore, the idea presented by Astolfi et al. is further extended to systems with nonlinear stiffness. In this work a reduced order observer using the notion of an invariant manifold has been designed for a 2-DOF mass–spring–damper system to estimate the displacement and velocity of one of the masses. In addition a comparative study is presented with a very well known observer based on the off-line solution of the Riccati equation for systems with Lipschitz type nonlinearity.

The approach presented in this paper requires the existence of a manifold that is invariant and attractive (Astolfi, Ortega, & Venkatraman, 2010; Besancon & Ticlea, 2007; Karagiannis, Carnevale, & Astolfi, 2008; Khan & Dhaouadi, 2015; Morbidi, Mariottini, & Prattichizzo, 2010). The manifold is made invariant by a nonlinear filter and attractive by proper selection of mapping functions. To prove the validity of the proposed observer, it is compared with a very well known nonlinear observer based on Lipschitz type non-linearity presented in Raghavan and Hedrick (1994), which is based on the off-line solution of the Riccati equation. The reason for this comparison is that the system under consideration has a Lipschitz type nonlinearity. The result is that both observers show satisfactory results under normal conditions, but the proposed new observer is more robust to parameter variation and phase change in the excitation signal. Finally the proposed reduced order observer is tested in a closed loop with a hybrid active and semi-active controller to demonstrate the practical utility of the technique.

The details of the proposed observer design are given in Section 2. In Section 3 we introduce the example system that will be used throughout this paper. The proposed observer design is applied to the example system in Section 4. In Section 5 an observer based on Lipschitz type nonlinearity is designed for the same example system. Comparison results for both observers are given in Section 6. In Section 7 the experimental system is described and then the experimental results are presented, followed by further discussion in Section 8.

2. Proposed observer design methodology

Consider a nonlinear, time-varying system described as

$$\dot{\eta} = f(\eta, y, t), \tag{1}$$

$$\dot{y} = h(\eta, y, t), \tag{2}$$

where $\eta \in \mathbb{R}^n$ is the unmeasured state, $y \in \mathbb{R}^m$ is the measurable output, an over-dot represents differentiation with respect to time, $f(\eta, y, t)$ and $h(\eta, y, t)$ are nonlinear functions. It is assumed that $f(\eta, y, t)$ and $h(\eta, y, t)$ are forward complete, i.e. trajectories starting at time t_0 are defined for all times $t \geq t_0$.

Let $\hat{\eta} \in \mathbb{R}^p$ represent the observer state, and $p \geq n$. From this, the total number of states of the system is $p + n$.

Then the dynamical system

$$\dot{\hat{\eta}} = \alpha(\hat{\eta}, y, t), \tag{3}$$

is called an observer for the system (1)–(2), if there exist mappings

$$\beta: \mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^p, \ \phi: \mathbb{R}^n \to \mathbb{R}^p,$$

with ϕ left invertible, such that the manifold

$$\mathcal{M}_t = \left\{ \left(\eta, y, \hat{\eta}\right) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p : \beta\left(\hat{\eta}, y, t\right) = \phi\left(\eta\right) \right\}, \tag{4}$$

has the following properties (Astolfi et al., 2007):

1. All trajectories of the extended system (1)–(3) that start on the manifold $\mathcal{M}_t$ at time t remain there for all future times, $\tau > t$ i.e. $\mathcal{M}_t$ is positively invariant.
2. All trajectories of the extended system (1)–(3) that start in a neighborhood of $\mathcal{M}_t$ asymptotically converge to $\mathcal{M}_t$.