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Spatial observer-based repetitive controller: An active disturbance rejection approach

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ABSTRACT

Linear Repetitive Control has proven to be an effective strategy to compensate for periodic disturbances in mechatronic systems that operate at constant speed; however, it renders very poor performance in varying speed applications. In this work, a Repetitive Controller based on a Generalized Proportional Integral (GPI) observer under Active Disturbance Rejection approach is presented and formulated in spatial domain. The inclusion of the linear GPI observer makes possible to see the spatial non-linear system as a simplified linear one by means of an on-line estimated unified disturbance term. Experimental results show that the presented linear approach successfully rejects periodic disturbances under varying speed conditions.

1. Introduction

Mechatronic rotary systems are exposed to many kinds of disturbances. Due to the nature of the these systems, periodic disturbances are one of the most common type of disturbances. They appear mainly because of eccentricities, axis unbalance, mass non-uniformity, couplings or pulsating torques.

To deal with the periodic disturbance rejection problem, well established control strategies as Repetitive Control (RC) (Wang, Gao, & Doyle, 2009) and Adaptive Feedforward Cancelation (AFC) (Messner & Bodson, 1994) have proven to be very effective. These strategies are based on the Internal Model Principle (IMP) (Francis & Wonham, 1976), which states that in order to track/reject an exogenous signal, the model of such signal must be included in the control loop. Both RC and AFC assume the exact knowledge of the signal frequency since this information is included in the internal model of the signal. If the mechatronic system speed remains constant the frequency of the disturbance is also constant and the above-mentioned techniques can be applied successfully. However, if the rotational speed changes, the frequency would change proportionally which causes that RC drastically loses its performance (Chew & Tomizuka, 1990; Steinbuch, 2002). To allow RC operate properly at varying speed, some modifications can be made: (1) Include an adaptive system in which the frequency of the internal model varies according to the signal frequency (speed). In this way, the system needs a frequency estimator and becomes a variable structure system (Hu, 1992; Tsao, Qian, & Nemani, 2000) which complicates the stability analysis. (2) Employ high order internal models to provide robustness against frequency changes, which is known as High Order Repetitive Control (HORC) (Steinbuch, Weiland, & Singh, 2007). The main drawbacks of HORC are that the order of the controller is very large and only small frequency changes are allowed. (3) Implement a digital system that adjusts the sampling frequency according to the speed changes (Olm, Ramos, & Costa-Castelló, 2010), in order to keep constant the number of samples per period of the disturbance signal. This allows larger frequency changes but involves a more complex stability analysis since the control system is a Linear Time Varying system (Olm, Ramos, & Costa-Castelló, 2011).

All above-mentioned strategies are formulated in time domain; however, the Spatial Repetitive Control (SRC) approach presented in Chen and Yang (2007) uses the angular position instead of time as the independent variable. The main idea behind spatial RC is that the disturbances generated in mechatronic systems are position dependent disturbances (those coming from eccentricities, axis unbalance, mass non-uniformity, coupling torques, etc.). Thus, the rotation of the mechanisms generates a disturbance that in the time domain has a frequency that varies proportionally with angular speed but in the spatial domain the frequency of disturbance remains invariant. As a consequence, a RC strategy in which the frequency of the disturbance is assumed fixed can be applied naturally if the position domain is used instead of time. An implication of this change of variable is that the system representation is now a nonlinear one which augments the complexity of the controller design. In general, a feedback linearization technique is employed and once a lineal model is obtained the lineal RC can be applied as usual (Quan & Cai, 2010). However, feedback linearization usually needs to be complemented with an additional robustifying technique in order to facilitate its practical implementation. Adaptive techniques as described in Chen and Yang (2009) and

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Yang and Chen (2011) are used with this purpose. These nonlinear techniques increase importantly the complexity of the control system. Furthermore, the control platform requires a spatial clock, usually an incremental encoder, to run the algorithm in spatial domain which yields a more complex control structure compared with its time domain counterpart.

In this paper a spatial observer-based repetitive controller for mechatronic systems is presented. The strategy is founded on the Proportional Integral Generalized (GPI) observer-based control approach (Sira-Ramírez, Luviano-Juárez, & Cortés-Romero, 2011) which is in line with the Active Disturbance Rejection (ADR) proposal (Radke & Gao, 2006; Tian & Gao, 2009). In this methodology, exogenous disturbances, unmodeled dynamics, non-linearities and parameter uncertainties are grouped in an unified disturbance term.

In the proposal presented here, a completely linear strategy is formulated which can deal with the nonlinear system and periodic disturbances in a robust way. The linear nature of the controller provides by itself a high level of simplicity compared with the above-mentioned strategies. In this way, the nonlinear dynamic components of the position domain plant representation are estimated on-line using a GPI observer. This estimation is then added to the control law, thus simplifying the controller design to a linear feedback strategy that uses the IMP to achieve rejection of periodic disturbances. The resulting control scheme requires a well defined number of parameters to be tuned through a simple algebraic structure. The control system design has two stages: firstly a GPI observer is designed with enough bandwidth to compensate for system non-linearities, this design is based on constructing a Luenberger-type observer for an extended model plant. Secondly, the RC is added to the control loop where the parameters needed to establish performance and stability are tuned in an algebraic fashion.

GPI controllers have been successfully tested in mechatronic systems (Luviano-Juárez, Cortés-Romero, & Sira-Ramírez, 2009) as well as in induction motor control (Cortés-Romero, Luviano-Juárez, & Sira-Ramírez, 2010). Here we use this technique in spatial domain combined with the rejection of periodic disturbances in a rotary mechatronic control system application in a discrete position framework. In this way, the presented control system simplifies the above-mentioned existing strategies since this constitutes a completely linear design and does not need any adaptive mechanism or frequency estimation.

This paper is organized as follows: Section 2 describes the plant system model, its spatial domain representation and a simplified linear model expression. Section 3 presents the GPI observer for the obtained model. In Section 4 the RC based on the GPI observer is described and also a brief explanation of the conventional RC is included. Section 6 shows the experimental results and conclusions are presented in Section 7. Finally, a prove of the Theorem 1, related to the observer error estimation bound, is presented in the Appendix.

2. Spatial domain system representation

This section presents the system transformation from time domain to spatial domain. The proposed methodology is applicable to SISO linear and nonlinear differentially flat systems of any order (see Sira-Ramírez & Agrawal, 2004).

2.1. General model

A general system model can be represented by the following n-order differential equation:

$$\frac{d^n}{dt^n}y(t)+a_{n-1}\frac{d^{n-1}}{dt^{n-1}}y(t)+\cdots+a_1\frac{d}{dt}y(t)+a_0y(t)+\Phi_t=b_0u(t), \tag{1}$$

where $u(t)$ is the system input, $y(t)$ is the flat system output and Φ_t are external disturbances and nonlinearities that depend on $t,\frac{d^{n-1}}{dt^{n-1}}y(t),\ldots,\frac{d}{dt}y(t),y(t)$.

As described in Chen and Yang (2009), the relation between time and space results

$$\theta=f(t)=\int_0^t\omega(\tau)\,d\tau+\theta(0),$$

with θ the angular position in revolutions and $\omega(t)$ the angular speed in rev/s. The following condition must be accomplished:

$$\omega(t)=\frac{d\theta}{dt}>0,$$

in order to assure the existence of the inverse function $t=f^{-1}(\theta)$.

Thus, the variables defined in time and space domain are related by[1]

$$\overline{x}(\theta)=x(f^{-1}(\theta)).$$

Therefore, the transformation from the system in time domain to spatial domain is based on the definition of the derivative term:

$$\frac{d}{dt}x(t)=\frac{d\theta}{dt}\frac{d\overline{x}(\theta)}{d\theta}=\overline{\omega}(\theta)\frac{d\overline{x}(\theta)}{d\theta}, \tag{2}$$

thus, using $\overline{y}(\theta)=\overline{\omega}(\theta)$ and definition (2), an expression for each derivative term in (1) can be found to be

$$\frac{d^k}{dt^k}y(t)=\overline{y}^k(\theta)\frac{d^k\overline{y}(\theta)}{d\theta^k}+\varsigma_k, \tag{3}$$

with $k=1,\ldots,n$, where ς_k groups nonlinear terms with lower order derivatives, having for the first three derivatives:

$$\varsigma_1=0,$$

$$\varsigma_2=\overline{y}(\theta)\left(\frac{d\overline{y}(\theta)}{d\theta}\right)^2,$$

$$\varsigma_3=4\overline{y}^2(\theta)\frac{d^2\overline{y}(\theta)}{d\theta^2}\frac{d\overline{y}(\theta)}{d\theta}+\overline{y}(\theta)\left(\frac{d\overline{y}(\theta)}{d\theta}\right)^3.$$

The nonlinear terms unified in Φ_t can be transformed into the spatial domain as $\overline{\Phi}_\theta$ using the same derivative transformation. Using these definitions, Eq. (1) results

$$\overline{y}^n(\theta)\frac{d^n\overline{y}(\theta)}{d\theta^n}+\varsigma_n+a_{n-1}\overline{y}^{n-1}(\theta)\frac{d^{n-1}\overline{y}(\theta)}{d\theta^{n-1}}+\varsigma_{n-1}$$
$$+\cdots+a_1\overline{y}(\theta)\frac{d\overline{y}(\theta)}{d\theta}+\varsigma_1+a_0\overline{y}(\theta)+\overline{\Phi}_\theta=b_0\overline{u}(\theta), \tag{4}$$

which yields

$$\frac{d^n\overline{y}(\theta)}{d\theta^n}=\frac{b_0}{\overline{y}^n(\theta)}\overline{u}(\theta)+\frac{1}{\overline{y}^n(\theta)}\Bigg(-\varsigma_n-a_{n-1}\overline{y}^{n-1}(\theta)\frac{d^{n-1}\overline{y}(\theta)}{d\theta^{n-1}}-\varsigma_{n-1}$$
$$-\cdots-a_1\overline{y}(\theta)\frac{d\overline{y}(\theta)}{d\theta}-\varsigma_1-a_0\overline{y}(\theta)-\overline{\Phi}_\theta\Bigg) \tag{5}$$

2.2. Simplified model

The variable change $\overline{v}(\theta)=\overline{u}(\theta)/\overline{y}^n(\theta)$, which constitutes a partial feedback linearization, allows obtaining a simplified linear model structure:

$$\frac{d^n\overline{y}(\theta)}{d\theta^n}=\kappa\overline{v}(\theta)+\overline{\xi}_1(\theta), \tag{6}$$

with $\kappa=b_0$ the system input gain and

$$\overline{\xi}_1(\theta)=\frac{1}{\overline{y}^n(\theta)}\Bigg(-\varsigma_n-a_{n-1}\overline{y}^{n-1}(\theta)\frac{d^{n-1}\overline{y}(\theta)}{d\theta^{n-1}}-\varsigma_{n-1}$$

[1] For the sake of clarity, the spatial-domain notations will be denoted by an upper bar.