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A jerk-constrained time-optimal servo with disturbance compensation

Yu-Sheng Lu, Raymond Shieh *

Department of Mechatronic Engineering, National Taiwan Normal University, Taipei 106, Taiwan

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ABSTRACT

For positioning servo-systems, this paper presents a jerk-constrained time- optimal control (JCTOC) scheme, augmented with an improved disturbance rejection method. In mechanical systems, the jerk that is the time derivative of acceleration may cause many unwanted results when too high. Thus, the JCTOC method is proposed to constrain the system's jerk and also obtain a time-optimal characteristic with the constrained jerk. However, because the JCTOC relies on the accuracy of the plant's model, system uncertainties and disturbances can adversely affect the output response. Thus, a disturbance observer (DOB) is added for compensation of the perturbation. The DOB used in this paper is in an integral form, and is thus referred to as an integral DOB (IDOB). The IDOB is further enhanced with a dynamic compensator to provide both better noise immunity and asymptotic compensation for disturbances of various orders.

1. Introduction

Positioning servos are widely employed in modern industry. While desiring to optimize response time, it is also important to have a smooth output response. Jerk, which is the time derivative of acceleration, also plays an important role in positioning servos. For example, a large jerk may cause unpleasant effects to elevator users, as well as vibrations, excitation of un-modeled dynamics, and high wear of mechanical parts. Hence, the maximum jerk should be limited to ensure smooth operation (Slaboda et al., 2003; Venkatesh, Cho, & Kim, 2002).

There are many studies on jerk limitation or reduction in the literature. For example, following the approach of Venkatesh et al. (2002), the jerk of an elevator should be restrained because of human sensitivity to changes in acceleration (jerk). Thus a trajectory profile of position is constructed, on which jerk limitation is imposed. According to Osornio-Rios, Romero-Troncoso, Herrera-Ruiz, and Castañeda-Miranda (2009), a high degree polynomial-based jerk-limited profile is applied to the operation of computer numerical control (CNC) machinery. In CNC machinery, small discontinuities in control efforts may cause excitation of high-frequency dynamics or actuator saturation. In contrast to the traditional trapezoidal velocity profile, this high degree polynomial-based jerk-limited profile has no control effort discontinuity problem. In the approach of Gasparetto and Zanotto (2008), the execution time and integral of the squared jerk are weighted and minimized in the trajectory planning. With a large weight on the jerk, the reference trajectory will be smooth but slow, whereas with a larger weight on time, the reference trajectory will be faster but less smooth. In the approach of Olabi, Béarée, Gibaru, and Damak (2010), the method of trajectory planning is used for controlling a six-axis machining robot, by which a jerk constraint is considered in the motion command generation. With this jerk-limited profile, endpoint vibration can be reduced, and residual vibration can be totally suppressed in some cases. Overall, these methods of jerk limitation in the literature are only concerned with trajectory-planning but lack a feedback system to confine the jerk. Thus, although the desired trajectory fulfills the jerk limitation, the system response may not satisfy the jerk limitation, especially when the system state is far from the desired trajectory. In the approach of Hoshijima and Ikeda (2007), the jerk of the so-called hand mass of a mechanical transfer system is reduced through feedback to suppress the vibration of the work mass. Although this method can effectively reduce system vibration through the system's coupling characteristics, it cannot directly constrain the jerk of the hand mass.

Unlike the path-planning approaches, a jerk-constrained time-optimal control (JCTOC) method (Shieh & Lu, 2010) can directly control the system jerk through the controller output, ensuring jerk restraint. However, that study does not provide detailed derivation of the JCTOC scheme, and more importantly, it only presents simulation results using an ideal plant, neglecting the effect of external disturbances and parametric errors that can be critical to the JCTOC. In this paper, a disturbance observer (DOB) is added to the system for correct mode switching and additional robustness. The DOB is a well-known disturbance compensation structure, which has been widely studied and applied to various systems ranging from motor drives to grinding mills (Grochmal & Lynch, 2007; Hace, Jezernik, & Sabanovic, 2007; Kakinuma, Sudo, & Aoyama, 2011; Kim, Kang, Kim, & Huh, 2009;

* Correspondence to: 162, Heping East Road Section 1, Taipei, Taiwan. Tel.: +886 2 7734 3512; fax: +886 2 2358 3074.
E-mail address: raymondlushieh@gmail.com (R. Shieh).

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Lin & Lee, 2011; Lu, 2008; Mohammadi, Tavakolib, Marquezb, & Hashemzadeh, 2013; Olivier, Craig, & Chen, 2012). Instead of using lowpass filters with a dc gain of unity in traditional DOBs, Chen, Tung, and Fuh (2008) and Tsai and Tung (2010) rearrange the DOB structure and incorporate an integrator into the DOB, here referred to as the integral DOB (IDOB). Although requiring less implementation effort, the IDOB cannot tune the gain at specific frequencies without affecting its bandwidth. In this paper, an advanced type of IDOB, referred to as the dynamically compensated IDOB (DC-IDOB), is proposed. The DC-IDOB is an enhanced modification of the conventional IDOB previously proposed (Chen et al., 2008; Tsai & Tung, 2010). By adding a dynamic compensator to the conventional IDOB, the DC-IDOB can have a transfer function of diverse characteristics. As an example, a disturbance model can be incorporated into the DC-IDOB, thus allowing the proposed IDOB to compensate for disturbances in specific forms more efficiently than the existing IDOB. In contrast to the previous study (Shieh & Lu, 2010), this paper describes both detailed derivation and experimental results of the JCTOC scheme. Moreover, the DC-IDOB is proposed in this paper and introduced to the JCTOC scheme for practical considerations. Experimental studies of both the JCTOC and the DC-IDOB are conducted to show the effectiveness of the control and the disturbance compensation methods.

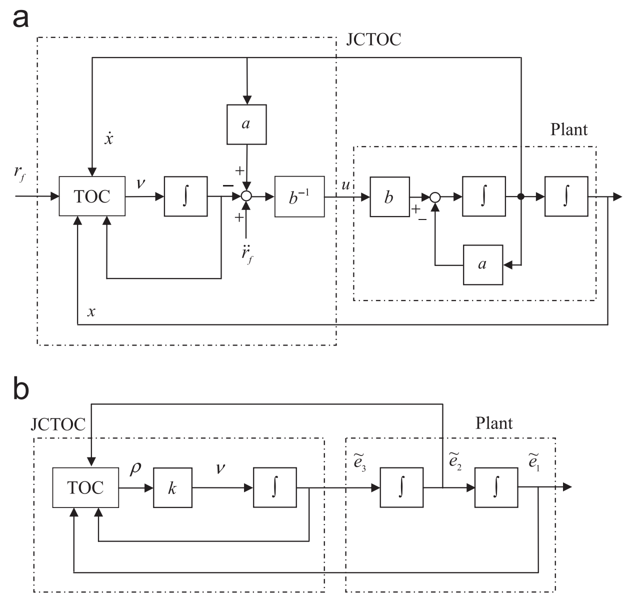

Fig. 1. Proposed JCTOC scheme. (a) Block diagram of JCTOC and (b) Equivalent block diagram.

2. Jerk-constrained time-optimal control (JCTOC)

In servo drives with a cascade control structure, the current control loops usually have much faster dynamics than the position control loop. Thus the dynamics of the current loops can be neglected in designing a position controller, and the motor drive can be considered a second-order system described by

$$\ddot{x} = -a\dot{x} + bu \tag{1}$$

where $x(t)$ is the displacement; $u(t)$is the control input, which is actually a torque-producing command to the current control loops; and a and b are plant parameters. Let $r_f(t)$ denote the position reference, and define the tracking error variables as $\tilde{e}_1(t) = r_f(t) - x(t)$, $\tilde{e}_2(t) = \dot{r}_f(t) - \dot{x}(t)$ and $\tilde{e}_3(t) = \ddot{r}_f(t) - \ddot{x}(t)$. The objective of the JCTOC is to track the position reference, $r_f(t)$, under the following constraint on the maximum magnitude of jerk: $|\dot{\tilde{e}}_3(t)| \le k$, where the constant k denotes the maximum admissible jerk.

Fig. 1(a) shows the block diagram of the JCTOC applied to the servomotor, in which the TOC denotes the non-linear time-optimal controller (Kaylon, 2002). Here the control input $u(t)$ generated by the JCTOC has an integral relationship to the output of the TOC, $\nu(t)$; that is

$$u(t) = b^{-1}\left\{ -\int \nu(t)dt + a\dot{x}(t) + \ddot{r}_f(t) \right\} \tag{2}$$

in which $\nu(t)$ is considered as a virtual control signal determined by the TOC. Examining the error dynamics of the system gives

$$\dot{\tilde{e}}_1(t) = \tilde{e}_2(t), \tag{3}$$

$$\dot{\tilde{e}}_2(t) = \tilde{e}_3(t), \tag{4}$$

$$\dot{\tilde{e}}_3(t) = \nu(t), \tag{5}$$

from which it can be seen $\nu(t)$ equals the jerk. Therefore, the JCTOC of the second-order system (1) with the maximum admissible jerk, k, becomes the TOC of a triple-integrator system (3)–(5) with the following virtual control input:

$$\nu(t) = k\rho(t), \tag{6}$$

in which $\rho(t)$ is a variable with the constraint: $|\rho(t)| \le 1$. By controlling $\rho(t)$ one can directly control the jerk of the plant, and having $|\rho(t)| \le 1$ implies fulfillment of the jerk constraint. But unlike the TOC of a triple-integrator plant, the motor-drive plant to be controlled by the JCTOC is only of second order. The JCTOC contains an artificially introduced integrator, and it is the output of this extra integrator that contributes to the actual control effort to the second-order plant, thus alleviating the bang-bang characteristics of the virtual control effort. By rearranging the block diagram shown in Fig. 1(a), the JCTOC method can be simplified to the form shown in Fig. 1(b). In this block diagram, the disturbance is neglected, and the JCTOC system with the addition of disturbance is considered in the following section. According to the system response by the time-optimal control (Akulenko & Kostin, 2000), define E_2 and E_1 as

$$E_2(\tilde{e}_3) = -\tilde{e}_3|\tilde{e}_3|/(2k), \tag{7}$$

$$E_1(\tilde{e}_2, \tilde{e}_3) = \frac{1}{k^2}\left\{ \frac{1}{3}\tilde{e}_3^3 - k\Delta^*\tilde{e}_2\tilde{e}_3 - \Delta^*\left[\frac{1}{2}\tilde{e}_3^2 - k\Delta^*\tilde{e}_2\right]^{3/2} \right\}, \tag{8}$$

in which $\Delta^* = -\mathrm{sgn}(\tilde{e}_2 - E_2)$, and $\mathrm{sgn}(\cdot)$ denotes the signum function. Let the switching surfaces be $\tilde{e}_1 - E_1 = 0$ and $\tilde{e}_2 - E_2 = 0$, thus giving the control law:

$$\rho = \begin{cases} \mathrm{sgn}(\tilde{e}_1 - E_1) & \text{if } \tilde{e}_1 - E_1 \ne 0 \\ \mathrm{sgn}(\tilde{e}_2 - E_2) & \text{if } \tilde{e}_1 - E_1 = 0,\ \tilde{e}_2 - E_2 \ne 0 \\ \mathrm{sgn}(\tilde{e}_3) & \text{if } \tilde{e}_1 - E_1 = 0,\ \tilde{e}_2 - E_2 = 0. \end{cases} \tag{9}$$

When the plant is initially at rest, it can be derived as shown in Appendix A, in which the switching instants of the JCTOC system are

$$t_{s1} = \left(\frac{r_f}{2k}\right)^{1/3}, \tag{10}$$

$$t_{s2} = 3t_{s1} = 3\left(\frac{r_f}{2k}\right)^{1/3}, \tag{11}$$

$$t_f = 4t_{s1} = 4\left(\frac{r_f}{2k}\right)^{1/3}, \tag{12}$$

where t_{s1} is the first switching instant, t_{s2} is the second switching instant, and t_f is the final time.

To avoid the singularity problem around the origin of the error state space, an additional switching plane is introduced. The