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New Damped-Jerk trajectory for vibration reduction

Richard Béarée

Arts et Métiers ParisTech, LSIS, 8 Bd Louis XIV, 59046 Lille, France

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ABSTRACT

This paper derives a jerk-shaped profile to address the vibration reduction of underdamped flexible dynamics of motion system. The jerk-limited profile is a widespread smooth command pattern used by modern motion systems. The ability of the jerk-limited profile to cancel the residual vibration of an undamped flexible mode is clearly explained using an equivalent continuous filter representation and the input shaping formalism. This motivates the design of a new jerk-shaped profile, named Damped-Jerk profile, to extend the previous result to the more common case of underdamped systems. Both simulations and experimental results demonstrate the effectiveness of the proposed Damped-Jerk profile to reduce damped vibration.

1. Introduction

The jerk-limited profile is a widespread trajectory pattern used by modern motion systems, such as mobile robots, machine-tools or industrial robots. Fig. 1 depicted such a profile, which is a time optimal solution to the problem of jerk limited rigid body control. The jerk limitation (the slope or time derivative of acceleration) is classically considered as a smoothness parameter, which is empirically used for the limitation of deformations and vibrations induced by the reference trajectory.

It is recognized that if the main vibration is not caused by external disturbances and if the flexible dynamics of the system is known, reference trajectory can be generated to cancel or reduce the vibration. One solution consists in defining a suitable parameterized trajectory, which assures that no oscillation occurs during and at the end of the motion. Such a method is mainly based on a system inversion principle. In Piazzi and Visioli (2000), this method appears to be very effective in reducing the residual vibration, and the smoothness of the control input makes it inherently robust to modeling errors. In Kim and Agrawal (2006), both the simulation and experimental results are presented to show the effectiveness of various jerk limited control profiles to reduce the vibration of a flexible structure. The authors state that the jerk limited rigid-body control profile such as versine or polynomial profile is effective to reduce the excitation of the flexible modes, however, better performance can be obtained from the handling of the low frequency modes as in an input shaped command. Input shaping is another known command generation methodology to suppress vibration, as detailed in Smith (1957), Singer and Seering (1990), Singhose and Pao (2009) and La-orpacharapan and Pao (2005). Input shaping relies on the convolution of a sequence of impulses with the reference signal. This convoluted signal is then used as a new reference for the system. Hence, vibrations induced by the first impulse is canceled or reduced at the end of the sequence. Input shaping has been implemented with success on numerous systems, and more specifically on manipulators and gantry cranes (Diaz, Pereira, Feliu, & Cela, 2010; Park, Chang, Park, & Lee, 2006; Peng, Singhose, & Frakes, 2012).

In Olabi, Béarée, Gibaru, and Damak (2010) and Béarée and Olabi (2013), we demonstrate that the jerk-limited profile can be used to cancel the residual vibration of an undamped system. The jerk time (i.e. the acceleration slope time) can be explicitly linked to the vibration caused by a dominating flexible mode. Hence, the jerk-limited profile can be seen as a special combination of the smoothness property of polynomial profiles and the vibration reduction property of basic input shaping. However, these elegant properties vanish when the damping coefficient of the considered flexible mode is not negligible. In this paper, we propose a simple modification of the jerk-limited profile in view of generalizing the previous property to the case of underdamped flexible mode. The resulting profile keeps the generic aspect, the easiness of implementation and the simplicity of tuning of the original jerk-limited profile, which are the main motivations of this study.

The outline of this paper is as follows: In Section 2 we present clear explanation regarding the ability of the jerk-limited profile to cancel the residual vibration for an undamped flexible mode and show how this profile can be characterized using input shaping

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E-mail address: richard.bearee@ensam.eu

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and continuous time filter formalism. We then proposed, in Section 3, a modified Damped-Jerk profile. In this section, the new profile is analyzed and compared to the original jerk-limited profile. In Section 4, experimental measurements conducted on an industrial robot confirm that the Damped-Jerk profile is an effective and elegant solution to the problem of underdamped vibration reduction.

2. Jerk-limited trajectory and vibration reduction

2.1. Input shaping formalism adapted to the jerk-limited profile

As depicted in Fig. 1, a jerk-limited profile implies a trapezoidal or a triangular acceleration profile. Such a trajectory can be easily and efficiently synthesized by applying a moving averaging FIR filter to an acceleration limited profile (sometimes called bang-off-bang profile), as described in Biagiotti and Melchiorri (2012) and Singh (2004). The filter time, equivalent to the jerk time noted T_J, is fixed by the relation between the maximum acceleration value and the maximum jerk value given by

$$T_J = A_{max}/J_{max}. \tag{1}$$

In the context of the trajectory following with kinematic constraints on acceleration and velocity, the filter introduces a time delay, which could have a detrimental effect on the accuracy of the new reference trajectory. In Béarée and Olabi (2013) a simple adaptation of the initial acceleration-limited trajectory is proposed to take account of the filter time. One notes that this methodology reduces significantly the algorithm complexity of a classical jerk-limited trajectory generator. Hence, a jerk-limited profile can be seen as the convolution of a simple averaging filter, noted F_{JL}, with an adapted acceleration-limited profile. In the same manner, the averaging filter can be perceived as the convolution of an integrator with a succession of two impulses given in continuous time domain by the transfer function

$$F_{JL}(s) = \frac{1}{s}(A_1 + A_2 e^{-sT_J}), \tag{2}$$

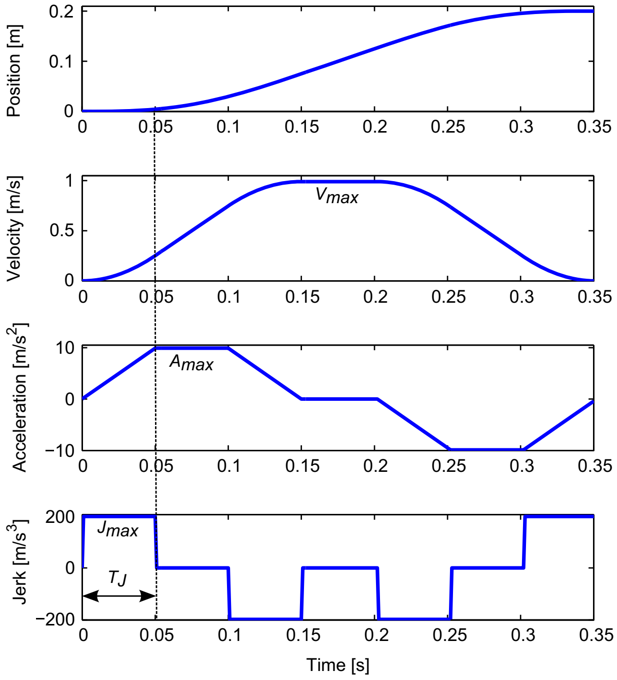

Fig. 1. Example of a jerk-limited rest-to-rest profile.

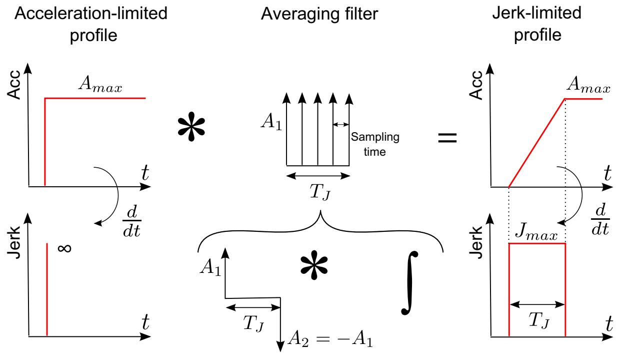

Fig. 2. Smooth input shaper associated to the jerk-limited profile.

with $A_1 = -A_2 = 1/T_J$. One notes that for discrete time implementation, T_J will be an integral multiple of the sampling interval.

Finally, as depicted in Fig. 2, the jerk-limited profile can be interpreted as a special case of a smooth negative input shaper. The smoothness is given by the low-pass filtering effect of the integrator and the negative input shaper part is the key explaining the vibration reduction ability of the jerk-limited profile. Considering a system submitted to a dominant undamped flexible mode at a frequency of ω_1, the cancellation of the residual vibration by the jerk-limited profile is equivalent to the zeros placement at the undamped flexible poles locations, i.e. at $s = \pm j\omega_1$. Applying this principle to the previous jerk shaper given by (2), the resulting equation to be solved becomes

$$F_{JL}(s)|_{s=\pm j\omega_1} = 0. \tag{3}$$

This expression can be rewritten as a system of trigonometric equations

$$\begin{aligned} &A_1 + A_2 \cos(\omega_1 T_J) = 0 \\ &\sin(\omega_1 T_J) = 0 \end{aligned} \tag{4}$$

A trivial solution to (4) is then given by

$$\begin{aligned} &T_J = k\frac{2\pi}{\omega_1} \\ &A_1 + A_2 = 0 \end{aligned} \tag{5}$$

with k being a positive integer. To keep the constraint imposed on the initial acceleration profile, the integral of the shaper has to be equal to 1, which conduct to the following set of solutions:

$$\begin{aligned} &T_J = k\frac{2\pi}{\omega_1} \\ &A_1 = -A_2 = \frac{1}{T_J} \end{aligned} \tag{6}$$

Based on the input shaping formalism, the previous result concludes the explanation about the capacity of jerk-limited to cancel the residual vibration for an undamped flexible mode. The jerk time can be taken equal to a multiple integer of the natural period of vibration. For time minimization, the jerk time is classically chosen equal to the natural period of the dominating flexible mode. Using this tuning methodology the maximum jerk value will be imposed by the jerk time and the maximum acceleration value according to the relation (1). The vibration reduction ability of such a strategy is sensitive to modeling errors or uncertainty on the frequency of the flexible mode. To clearly measure this effect, the amplitude of residual vibration can be plotted as a function of modeling errors. Such a plot is classically called a sensitivity curve.