Control Engineering Practice 21 (2013) 1553–1563

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Discrete dynamic optimization of N-stages control for isolated signalized intersections

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ARTICLE INFO

Article history:
Received 20 January 2012
Accepted 25 July 2013
Available online 22 August 2013

Keywords:
Traffic signal control
Discrete-event systems
Optimization
Optimal control
Transient analysis

ABSTRACT

The N-stages control problem for isolated signalized intersections is defined as the control problem to disperse initial queue lengths to their optimal steady-state values in N cycles. Based on a discrete-event model of a simplified isolated signalized intersection, the N-stages control problem is formulated as a linear programming problem as well as a quadratic programming problem. A new algorithm is proposed for solving the discrete optimization problem by simple calculations, based on the optimal solution of the corresponding continuous-time problem. Numerical comparisons between the continuous-time optimal solution and the discrete-event optimal solutions, obtained from linear programming and sequential quadratic programming, are given for a few examples.

1. Introduction

Traffic congestion is a common problem in most urban cities all over the world. Congestion has several effects on travelers, businesses, agencies, and cities. One significant element is the value of the additional time and wasted energy. The congestion in USA's cities areas is increasing continuously, e.g. in 2011 congestion cost (based on wasted time and fuel) was about $121 billion (Schrank, Eisele, & Lomax, 2012), up by one billion dollars from the year before. Of that total, about 27 billion worth was wasted time and diesel fuel from trucks moving goods on the network.

Efficient traffic signal control at signalized intersections can improve mobility, prevent queue spillbacks, and relieve congestion in cities. While this paper deals with a control level of isolated intersections, other traffic control strategies for a higher or network level of urban regions, i.e. considering urban area with several intersections, can be found in Geroliminis, Haddad, and Ramezani (2013) and Haddad, Ramezani, and Geroliminis (2013). The control of signalized intersections in urban transportation systems is a problem of great importance that has attracted a lot of attention and has been investigated by many researchers during the last few decades. Recently, with the introduced concept of perimeter control for urban regions in e.g. Geroliminis et al. (2013), i.e. manipulating the traffic flows that enter and exit an urban region, the control of isolated intersections becomes challenging as they are the traffic measures that can actuate the perimeter control decisions in hierarchical control schemes.

The first optimal control policy for the queue dispersal of oversaturated intersections was proposed in Gazis and Potts (1963) and Gazis (1964), derived by the Pontryagin maximum principle. The policy was meant to minimize the total delay under the constraint of complete and simultaneous dispersal of the queue lengths in all approaches (Ioslovich, Gutman, & Borshchevsky, 2011; Ioslovich, Haddad, Gutman, & Mahalel, 2011). The proposed model was a continuous-time model with the assumption that the cycle length is fixed over the rush period, defined as the time period that starts when queues would develop and be maintained and ends at the earliest time that queues can be dissolved simultaneously. This model was also used by other researchers (Guardabassi, Locatelli, & Papageorgiou, 1984; Michalopoulos & Stephanopoulos, 1977a, 1977b, 1978).

The structure of the optimal steady-state traffic control is revealed in Haddad, De Schutter, Mahalel, Ioslovich, and Gutman (2010) where it is shown that the final reachable queue lengths at the end of the rush period, i.e. at steady-state, are not equal to zero, as assumed in Gazis and Potts (1963) and Gazis (1964). The optimal steady-state solution for an extended model with lost time and constraints on the green durations is given in Haddad, Mahalel, Ioslovich, and Gutman (2010). Using the Pontryagin maximum principle, the optimal control policy is derived in Ioslovich, Haddad et al. (2011) without the simultaneous queue dissipation assumption. The optimal synthesis of green light split in an isolated intersection for all cases of initial queues and green split bound conditions is demonstrated.

The above mentioned models (Gazis, 1964; Gazis & Potts, 1963; Guardabassi et al., 1984; Michalopoulos & Stephanopoulos, 1977a, 1977b, 1978) are continuous-time, while in fact, discrete-event models, e.g. Haddad, De Schutter et al. (2010) and Chang and Lin

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0967-0661/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.conengprac.2013.07.007

(2000), are closer to reality. In the continuous-time model, the "switch-over time", defined in Gazis and Potts (1963) and Gazis (1964), for an intersection with two directions as the time instant when the service rate is switched from its maximum to its minimum for the direction associated with the maximum saturation flow, or the end of the rush period may occur not at the end of a cycle, while these two time instants in a discrete-event model coincide exactly with the end of a cycle.

A discrete minimal delay model to solve the oversaturated isolated intersection problem is proposed in Chang and Lin (2000). The queue lengths and delays during a cycle are calculated. The discrete Hamiltonian approach is used and the solution is "bang-bang" control consistent with the Pontryagin maximum principle solution in Gazis (1964). It is assumed in the discrete minimal delay model that no queue at any approach becomes negative or zero before the end of the rush period, i.e. all the queues are dissolved simultaneously. The model cannot describe a positive zero-queue length period (ZQLP), defined in Haddad, De Schutter et al. (2010) as the time period for which the queue length is equal to zero while the signal is green. The algorithm in Chang and Lin (2000) adjusts the adjoint variables iteratively and terminates when the queue length of each movement is negative at the end of its green light. No attempt is made to reach optimal steady-state queue lengths.

The discrete-event model in the current paper is essentially the same as in Haddad, De Schutter et al. (2010). The queue lengths are constrained to be non-negative and *non-increasing* from one cycle to the next cycle, and the final queue lengths are the optimal steady-state queue lengths, defined in Haddad, De Schutter et al. (2010), Haddad, Mahalel, De Schutter, Ioslovich, and Gutman (2009), and Haddad, De Schutter, Mahalel, and Gutman (2009). The optimal discrete N-stages problem is defined as the optimal control problem to disperse initial queue lengths to their optimal steady state values in N cycles, and is formulated both as a linear, and as a quadratic programming problem. Moreover, an algorithm is proposed to obtain the optimal solution for the N-stages control without solving the problem by linear or quadratic programming, based on the optimal policy principles obtained from the optimal control solution for the continuous-time model in Gazis and Potts (1963), Gazis (1964), Michalopoulos and Stephanopoulos (1977a, 1978), and Ioslovich, Haddad et al. (2011).

This paper is organized as follows. First, the problem of N-stages control is defined in Section 2. The discrete-event model and the optimization problem are introduced in Section 3, while the continuous-time model calculations are given in Section 4. In Section 5 the algorithm for solving the discrete optimization are developed, and some case study examples are presented in Section 6, which is followed by discussion and conclusions.

2. Problem definition of N-stages control

Let us consider a simplified isolated vehicular traffic intersection with two one-way movements (m_1 and m_2), defined as the sets of vehicles having reached but not passed the intersection.[1] Each movement is governed by a traffic signal that each can be either green or red. Since the two movements cannot occupy the intersection area simultaneously, the traffic signals will be opposite, i.e. when movement m_1 has green light, movement m_2 sees red light, and vice versa. Each movement will encounter intertwined green and red periods. Without loss of generality the amber period is not explicitly considered, and a cycle is defined as a pair of one green and one red period, whose durations may be time-varying. The queue length for a movement is defined as the number of vehicles belonging to the movement which is behind the stop line, i.e. the queue does not include the vehicles that are inside the intersection or have passed it.

For the isolated signalized intersection, we intend to determine the N-stages traffic signal control solution that brings initial oversaturated queue lengths to their optimal steady-state values in N cycles while minimizing a given queue length dependent criterions, under green duration and cycle constraints. We intend to formulate linear and quadratic programming problems to obtain the optimal solution. Note that the continuous-time problem, which does not consider the queue evolution within the cycle and ignores the green-red switching, was investigated in Ioslovich, Haddad et al. (2011) where the optimal control solutions were derived and numerical comparisons with other earlier solutions in the literature were presented. We also intend to propose an algorithm to obtain the optimal solution for the N-stages control without solving the problem by linear or quadratic programming, based on the optimal policy principles obtained from the optimal control solution for the continuous-time model.

3. A discrete-event model for isolated signalized intersections

In this paper, the model of the queue dynamics at a simplified isolated signalized intersection with two movements m_1 and m_2, without lost time and green duration constraints, is the same as in Haddad, De Schutter et al. (2010), Haddad, De Schutter et al. (2009), and Haddad, Mahalel et al. (2009), see Fig. 1. The relaxed discrete-event max-plus (R-DMP) problem defined in Haddad, De Schutter et al. (2010)[2] is used to solve the discrete optimal problem for N-stages control when the criterion J is a strictly increasing function of the queue lengths[3]

$$\min_{\substack{g_1(0),g_2(0),g_1(1),g_2(1),\ldots,g_1(N-1),g_2(N-1),\\ q_1(t_1),q_2(t_2),q_1(t_3),q_2(t_4),\ldots,q_1(t_{2N-1}),q_2(t_{2N})}} J \tag{1}$$

subject to

$$q_1(t_{2k+1}) \geq q_1(t_{2k}) + (a_1(t_{2k}) - d_1(t_{2k})) \cdot g_1(k) \tag{2}$$

$$q_1(t_{2k+1}) \geq 0 \tag{3}$$

$$q_1(t_{2k+2}) = q_1(t_{2k+1}) + a_1(t_{2k+1}) \cdot g_2(k) \tag{4}$$

$$q_2(t_{2k+1}) = q_2(t_{2k}) + a_2(t_{2k}) \cdot g_1(k) \tag{5}$$

$$q_2(t_{2k+2}) \geq q_2(t_{2k+1}) + (a_2(t_{2k+1}) - d_2(t_{2k+1})) \cdot g_2(k) \tag{6}$$

$$q_2(t_{2k+2}) \geq 0 \tag{7}$$

$$q_1(t_{2k}) \geq q_1(t_{2k+2}) \tag{8}$$

$$q_2(t_{2k}) \geq q_2(t_{2k+2}) \tag{9}$$

$$q_1(t_0) = q_{1,\text{init}} \tag{10}$$

$$q_2(t_0) = q_{2,\text{init}} \tag{11}$$

$$q_1(t_{2N}) = q_{1,\text{ss}} \tag{12}$$

[1] Note that the model can be extended for intersections with more than two movements, which is tedious but straightforward.

[2] The R-DMP problem neglects the lost time, however an R-DMP problem with lost time is equivalent with an DMP problem without lost time if time and arrival rates are scaled, see Ioslovich, Haddad et al. (2011).

[3] Let the queue length vector q be defined as $[q_1(t_1), q_1(t_2), q_2(t_1), q_2(t_2), \ldots, q_1(t_{2N-1}), q_1(t_{2N}), q_2(t_{2N-1}), q_2(t_{2N})]^{\mathrm{T}}$. The criterion function J is said to be a strictly increasing function of the queue lengths, if, for all queue length vectors $\hat{q}$, $\tilde{q}$ with $\hat{q} \leq \tilde{q}$ (elementwise) and $\hat{q}_i < \tilde{q}_i$ for at least one index i, it holds that $J(\hat{q}) < J(\tilde{q})$.