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Nonlinear region of attraction analysis for flight control verification and validation

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ABSTRACT

Current practice for flight control validation relies heavily on linear analyses and nonlinear, high-fidelity simulations. This process would be enhanced by the addition of nonlinear analyses of the flight control system. This paper demonstrates the use of region of attraction estimation for studying nonlinear effects. A nonlinear polynomial model is constructed for the longitudinal dynamics of NASA's Generic Transport Model aircraft. A polynomial model for the short period dynamics is obtained by decoupling this mode from the nonlinear longitudinal model. Polynomial optimization techniques are applied to estimate region of attractions around trim conditions.

1. Introduction

Safety critical flight systems require extensive validation prior to entry into service. Validation of the flight control system is becoming more difficult due to the increased use of advanced flight control algorithms, e.g. adaptive flight controls. NASA's Aviation Safety Program (AvSP) aims to reduce the fatal (commercial) aircraft accident rate by 90% by 2022 (Heller, Niewoehner, & Lawson, 2003). A key challenge in achieving this goal is the need for extensive validation and certification tools for the flight systems. Current certification and validation procedures involve analytical, simulation-based and experimental techniques (Heller et al., 2003). Current practice is to assess the closed-loop stability and performance characteristics of the aircraft flight control system around numerous trim conditions using linear analysis tools. The linear analysis methods include stability margins, robustness analysis and worst-case analysis. The linear analysis results are supplemented with Monte Carlo simulations of the full nonlinear equations of motion to provide further confidence in the system performance and to uncover nonlinear dynamic characteristics, e.g. limit cycles. Hence, current practice involves extensive linear analysis at different trim conditions and probabilistic nonlinear simulation results. The certification process typically does not involve analytical nonlinear methods.

The gap between linear analyses and Monte Carlo simulations can cause significant nonlinear effects to go undetected. For example, several F/A-18 aircraft were lost due to a nonlinear loss-of-control phenomenon known as the falling leaf mode (Heller, Niewoehner, & Lawson, 1999; Heller, David, & Holmberg, 2004; Jaramillo & Ralston, 1996; Lluch, 1998). Linear analysis tools did not detect the potential of the closed-loop system to exhibit the falling leaf mode. Thus there is a need for nonlinear analysis tools to fill this gap (Chakraborty et al., 2010). Recently, significant research has been performed on the development of nonlinear analysis tools for computing regions of attraction, reachability sets, input–output gains, and robustness with respect to uncertainty for nonlinear polynomial systems (Chiang & Thorp, 1989; Davison & Kurak, 1971; Genesio, Tartaglia, & Vicino, 1985; Parrilo, 2000; Tan, 2006; Tan, Topcu, Seiler, Balas, & Packard, 2008; Tibken, 2000; Tibken & Fan, 2006; Topcu, Packard, Seiler, & Wheeler, 2007, 2008; Vannelli & Vidyasagar, 1985). These tools make use of polynomial sum-of-squares (SOS) optimization (Parrilo, 2000). Unfortunately, the polynomial SOS techniques can only be applied to the dynamics described by polynomial vector field, though they offer great potential to bridge the gap in the flight control validation process.

The objective of this paper is to demonstrate the advantage of including nonlinear analysis tools based on SOS techniques in the flight control law validation process. The computational requirements for sum-of-squares (SOS) optimizations grow rapidly in the number of variables and polynomial degree. This roughly limits SOS methods to nonlinear analysis problems with at most 8–10 states and degree 3–5 polynomial models. This computational constraint does not limit the usefulness of these techniques. The construction of accurate, low-degree polynomial models is an important step in the proposed analysis process.

This paper applies the nonlinear analysis tools on NASA's Generic Transport Model (GTM) aircraft (Cox, 2009; Murch & Foster, 2007). The GTM is the primary test aircraft for NASA's

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Airborne Subscale Transport Aircraft Research (AirSTAR) flight test facility (Bailey, Hostetler, Barnes, Belcastro, & Belcastro, 2005; Jordan & Bailey, 2008; Jordan, Foster, Bailey, & Belcastro, 2006). The AirSTAR program addresses the challenges associated with validating flight control law in adverse condition (Gregory, Cao, Xargay, Hovakimyan, & Zou, 2009; Murch, 2008; Murch, Cox, & Cunningham, 2009). The polynomial model constructed in this paper accurately represents the longitudinal dynamics of NASA's Generic Transport Model (GTM) aircraft and it is suitable to address the issues with flight control law validation and verification.

The paper has the following structure. First, a polynomial model of the longitudinal dynamics of NASA's GTM aircraft (Cox, 2009; Murch & Foster, 2007) is constructed in Section 2. The longitudinal dynamics consist of a phugoid and short period mode. In Section 2.3, a polynomial model for the short period dynamics is obtained by decoupling this mode from the nonlinear longitudinal model. This nonlinear short period model is of interest because the decoupling of the longitudinal modes is typically done using linearized models. Section 3 describes a computational procedure to estimate regions of attraction for polynomial systems (Jarvis-Wloszek, 2003; Jarvis-Wloszek, Feeley, Tan, Sun, & Packard, 2003, 2005; Tan, 2006; Tan & Packard, 2004; Topcu et al., 2007, Topcu, Packard, & Seiler, 2008). This algorithm is applied in Section 4 to estimate regions of attractions for the open-loop short period dynamics and a closed-loop longitudinal GTM aircraft. Analysis of the 2-state short period model is presented in Section 4 is for illustrative purposes since the system trajectories can be entirely visualized in a phase-plane diagram. This model is used to demonstrate that the linearized model fails to capture significant nonlinear effects. The analysis of the 4-state longitudinal GTM aircraft demonstrates that the nonlinear region of attraction (ROA) computational procedure can be applied to systems with higher state dimensions. The paper concludes with a summary of the contribution of the paper.

2. Polynomial aircraft models

NASA's Generic Transport Model (GTM) describes a remote-controlled 5.5 percent scale commercial aircraft (Cox, 2009; Murch & Foster, 2007). The main GTM aircraft parameters are provided in Table 1. NASA constructed a high fidelity 6 degree-of-freedom Simulink model of the GTM with the aerodynamic coefficients described as look-up tables. This section describes the construction of polynomial models of the GTM longitudinal and short period dynamics based on the look-up table data.

2.1. Longitudinal dynamics

The longitudinal dynamics of the GTM are described by a standard 4-state longitudinal model (Stevens & Lewis, 1992):

$$\dot{V} = \frac{1}{m}(-D - mg\sin(\theta-\alpha) + T_x\cos\alpha + T_z\sin\alpha) \tag{1}$$

$$\dot{\alpha} = \frac{1}{mV}(-L + mg\cos(\theta-\alpha) - T_x\sin\alpha + T_z\cos\alpha) + q \tag{2}$$

Table 1
Aircraft and environment parameters.

Wing area, S	0.5483 m^2
Mean aerodynamic chord, $\overline{c}$	0.2790 m
Mass, m	22.50 kg
Pitch axis moment of inertia, I_{yy}	5.768 kg-m^2
Air density, ρ	1.224 kg/m^3
Gravity constant, g	9.810 m/s^2

$$\dot{q} = \frac{(M+T_m)}{I_{yy}} \tag{3}$$

$$\dot{\theta} = q \tag{4}$$

where V is the air speed (m/s), α is the angle of attack (rad), q is the pitch rate (rad/s) and θ is the pitch angle (rad). The control inputs are the elevator deflection δ_{elev} (rad) and engine throttle δ_{th} (percent). For ease of interpretation, plots of α, q and δ_{elev} are shown in units of deg, deg/s, and deg, respectively.

The drag force D (N), lift force L (N), and aerodynamic pitching moment M (N m) are given by

$$D = \overline{q}SC_D(\alpha,\delta_{elev},\hat{q}) \tag{5}$$

$$L = \overline{q}SC_L(\alpha,\delta_{elev},\hat{q}) \tag{6}$$

$$M = \overline{q}S\overline{c}C_m(\alpha,\delta_{elev},\hat{q}) \tag{7}$$

where $\overline{q} := \frac{1}{2}\rho V^2$ is the dynamic pressure (N/m^2) and $\hat{q} := (\overline{c}/2V)q$ is the normalized pitch rate (unitless). C_D, C_L, and C_m are unitless aerodynamic coefficients computed from look-up tables provided by NASA.

The GTM has one engine each on the port and starboard sides of the airframe. Equal thrust settings for both engines is assumed. The thrust from a single engine T (N) is a function of the throttle setting δ_{th} (percent). $T(\delta_{th})$ is a given ninth-order polynomial in NASA's high fidelity GTM simulation model. T_x (N) and T_z (N) denote the projection of the total engine thrust along the body x and body z-axes, respectively. T_m (N m) denotes the pitching moment due to both engines. T_x, T_z and T_m are given by

$$T_x(\delta_{th}) = n_{ENG}T(\delta_{th})\cos(\varepsilon_2)\cos(\varepsilon_3) \tag{8}$$

$$T_z(\delta_{th}) = n_{ENG}T(\delta_{th})\sin(\varepsilon_2)\cos(\varepsilon_3) \tag{9}$$

$$T_m(\delta_{th}) = r_zT_x(\delta_{th}) - r_xT_z(\delta_{th}) \tag{10}$$

$n_{ENG}=2$ is the number of engines. $\varepsilon_2 = 0.0375$ rad and $\varepsilon_3 = -0.0294$ rad are angles specifying the rotation from engine axes to the airplane body axes. $r_x=0.1371$ m and $r_z=0.0907$ m specify the thrust moment arm.

2.2. Polynomial longitudinal model

The following terms of the longitudinal model presented in Section 2.1 are approximated by low-order polynomials:

1. Trigonometric functions: $\sin(\alpha)$, $\cos(\alpha)$, $\sin(\theta-\alpha)$, $\cos(\theta-\alpha)$.
2. Engine model: $T(\delta_{th})$.
3. Rational dependence on speed: $1/V$.
4. Aerodynamic coefficients: C_D, C_L, C_m.

Constructing polynomial approximations of the trigonometric functions, engine model, and rational dependence on speed is relatively straight-forward. The trigonometric functions are approximated by Taylor series expansions: $\sin z \approx z - \frac{1}{6}z^3$ and $\cos z \approx 1 - \frac{1}{2}z^2$ for z in units of radians. For $|z| \le \pi/4$ rad the maximum approximation error for the sine and cosine functions is 0.35% and 2.2%, respectively. For the engine model, a least-squares technique is used to approximate the ninth order polynomial function $T(\delta_{th})$ by the following third order polynomial:

$$T(\delta_{th}) \approx -8.751 \times 10^{-6}\delta_{th}^3 + 5.115 \times 10^{-3}\delta_{th}^2 + 3.673 \times 10^{-1}\delta_{th} + 4.825 \tag{11}$$

The maximum approximation error is 1.3% over the full range throttle inputs $\delta_{th} \in [0\%, 100\%]$. The least-squares technique is also used to compute a linear fit to $1/V$ over the desired range of interest