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Implicit discrete-time twisting controller without numerical chattering: Analysis and experimental results

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A B S T R A C T

In this paper, we present an implementation of the sliding mode twisting controller on an electropneumatic plant for a tracking control problem. To this end, implicitly and explicitly discretized twisting controllers are considered. We discuss their structure, properties and implementations, as well as the experimental results. The analysis of the performance sustains the theoretical superiority of the implicitly discretized version, as shown in previous works. The main advantages of the implicit method are better tracking performance and drastic reduction in the input and output chattering. This is achieved without modifying the structure of the controller compared to its continuous-time version. The tracking error cannot be used as the sliding variable: it has a relative degree 3 w.r.t. the control input. The tuning of the sliding surface has well as some other parameters in the control loop was instrumental in achieving good performance. We detail the selection procedure of those parameters and their influence on the closed-loop behavior. Finally we also present some results with an implicitly discretized EBC-SMC controller.

1. Introduction

Implementation of control laws is almost exclusively done using microcontrollers. This implies that the controller is in discrete-time rather than in continuous-time. In sliding mode control, this can induce a degradation of the performance by contributing to the chattering phenomenon. We call this the *numerical chattering*. An intense activity over the last 30 years was devoted to the reduction of this numerical chattering, mainly for equivalent control based sliding mode control (ECB-SMC). In the early 90s, second order sliding mode control concept was introduced in Levant (1993) and sparked the development of a large wealth of literature. One of the first controllers of this kind was the twisting controller which features a discontinuous control action w.r.t. the sliding variables. However, to the best of our knowledge, few discrete-time versions of the twisting controller have been proposed. The substitution of the signum function by a saturation, common trick to reduce the chattering for first order SMC, has no straightforward extension to the twisting algorithm. It is then fair to assume that the explicit discretization was used to get a discrete-time twisting controller, like in Taleb, Levant, and Plestan (2013).

The other discretization method we consider is the implicit method. It has been used for a long time in the nonsmooth mechanics community, but it was not applied in control theory until very recently (Acary & Brogliato, 2010; Acary, Brogliato, & Orlov, 2012; Huber, Acary, & Brogliato,... 2013a, 2013b). The implicit discretization of the twisting controller was first studied in Acary et al. (2012). Roughly speaking, the difference between the explicit and implicit methods in our context is the following: given a partition $\{t_k\}$ of a time interval, with the *explicit* discretization, at the time instant t_k, the argument of the signum function is the value of the sliding variable at t_k, whereas with the *implicit* discretization it is the value at t_{k+1}. Despite its name and formulation, the implicitly discretized twisting controller is non-anticipative and induces a well-defined behavior, as we shall see in Section 2. Its main features are the drastic reduction of the output chattering and the reduction of the control input chattering, that is the control input is no more of the high frequency “bang-bang” type. In the discrete-time sliding regime, the control input is also insensitive to an increase of the gain. To simplify the nomenclature, we refer to the discrete-time twisting controller with an implicit (resp. explicit) discretization as the implicit (resp. explicit) twisting controller.

In the following, we present results from an implementation of both explicit and implicit twisting controllers on an electropneumatic plant. The control problem at hand is the tracking of a sinusoidal trajectory for the position of the end of the piston. The analysis of the gathered data supports the theoretically predicted reduction of the chattering claimed in Acary et al. (2012) and also the claim that the numerical chattering can be the main source of

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chattering, see Huber et al. (2013a). This highlights the importance of the discretization process which is unfortunately often overlooked both in the analysis and in the implementation.

The second part of the paper is dedicated to the choice of three parameters: the first one defines the sliding variable and the two others are constants for two filtered differentiators.

The influence of those parameters is only visible with the implicit controller. With an explicit one, the performance is not good enough to always see a change when their values change. It appears that with an implicit controller the differentiators become the weakest component in the control loop. Empirical data suggest that the three parameters have to be tuned simultaneously. To help with the tuning, we present the selection procedure that we used. We also analyze how the experimental tracking performance varies with the choice of the sliding surface. We hope that this presentation raises awareness for the importance of tuning to get the best possible performance for systems with similar setup.

In the remainder of this section, we introduce the notations. In Section 2 we briefly recall the twisting controller in continuous-time as well as in discrete-time. The experimental setup is presented in Section 3 as well as the control scheme. Then the experimental results are analyzed in Section 4. In Section 5, we deal with the tuning of some control parameters and the impact it has on the performance. In Section 6 an experimental comparison between the twisting and a classical first order SMC is proposed. Conclusions end the paper in Section 7.

Notations: The sliding variable is denoted by σ, it is supposed to be at least twice differentiable and Σ denotes $(\sigma\dot{\sigma})^T$. The control value changes at time instants t_k, defined as $t_k := t_0 + kh$ for all $k \in \mathbb{N}$ with $t_0, h \in \mathbb{R}_+$. The scalar h is called the sampling period. Let $\sigma_k := \sigma(t_k)$ and $\dot{\sigma}_k := \dot{\sigma}(t_k)$ for all $k \in \mathbb{N}$. The tilded variants $\tilde{\sigma}$, $\tilde{\dot{\sigma}}$ and $\tilde{\Sigma}$ denote variables used in the controller. Let sgn be the classical single-valued signum function: for all $x > 0$, $\mathrm{sgn}(x) = 1$, $\mathrm{sgn}(-x) = -1$ and $\mathrm{sgn}(0) = 0$.

Definition 1 (*Multivalued signum function*). Let $x \in \mathbb{R}$. The multivalued signum function $\mathrm{Sgn}\colon \mathbb{R} \rightrightarrows [-1, 1]$ is defined as:

$$\mathrm{Sgn}(x) = \begin{cases} \{1\} & x > 0 \\ \{-1\} & x < 0 \\ [-1, 1] & x = 0. \end{cases}$$

If $x \in \mathbb{R}^n$, then the vector-valued signum function $\mathrm{Sgn}\colon \mathbb{R}^n \rightrightarrows [-1, 1]^n$ is defined as $\mathrm{Sgn}(x) := (\mathrm{Sgn}(x_1), \ldots, \mathrm{Sgn}(x_n))^T$.

2. The twisting controller

2.1. Continuous-time twisting

The twisting algorithm was one of the first second-order sliding mode controllers presented in the literature Levant (1993). It requires the control input u to be of relative degree 2 with respect to the sliding variable σ, that is

$$\ddot{\sigma}(x, t) = a(x, t) + b(x, t)u, \tag{1}$$

with the following bounds: for all $(x, t) \in \mathbb{R}^n \times \mathbb{R}_+$,

$$0 \le K_m \le |b(x, t)| \le K_M \quad \text{and} \quad |a(x, t)| \le K_a. \tag{2}$$

The control law for the twisting controller is

$$u \in -r_1\,\mathrm{Sgn}(\sigma) - r_2\,\mathrm{Sgn}(\dot{\sigma}), \tag{3}$$

and with the conditions

$$\begin{cases} (r_1 + r_2)K_m - K_a > (r_1 - r_2)K_M + K_a \\ (r_1 - r_2)K_m > K_a, \end{cases} \tag{4}$$

the state of the closed-loop system (1) and (3) converges to the origin in finite time. The solutions of the closed-loop system are defined within Filippov's framework (Filippov, 1988). Lyapunov functions for this controller have been recently investigated, see Orlov (2005) and Polyakov and Poznyak (2009). In this paper, we follow the convention of using $G := r_1$ and $\beta := r_2/r_1$, instead of r_1 and r_2. The conditions listed in (4) impose that $0 < \beta < 1$.

It is worth noting that the controller (3) is by definition multivalued and that the control input u is a selection of the closed-loop differential inclusion formed by (1) and (3).

2.2. The two discrete-time twisting controllers

The control input obtained from a microcontroller is usually a step function, and its value is periodically updated. We model the control input function as $u(t) = u_k$ for $t \in (t_k, t_{k+1}]$. When implementing this controller, the task at hand at each time instant t_k is to select the control input value from all the possible values defined by a discretization of (1) and (3). We want the discrete-time version to keep the multivalued nature of the controller. This is achieved by using the implicit discretization, which applied in (3) gives

$$u_k \in -G\,\mathrm{Sgn}(\sigma_{k+1}) - \beta G\,\mathrm{Sgn}(\dot{\sigma}_{k+1}), \tag{5}$$

whereas the explicit discretization yields

$$u_k = -G\,\mathrm{sgn}(\sigma_k) - \beta G\mathrm{sgn}(\dot{\sigma}_k). \tag{6}$$

Note that the relation in (6) is not an inclusion since the right-hand side is a given singleton at time t_k. The case where either σ_k or $\dot{\sigma}_k$ is zero is clearly pathological. Hence the signum function in (6) is single-valued, contrarily to the continuous-time case. The computation of the control input value is in this case straightforward from (6).

With the implicit discretization, a discrete-time version of the dynamics (1) is required to perform the computation. We recast the closed-loop dynamics (1) and (5) as a first order system with state $\Sigma := (\sigma\dot{\sigma})^T$. In the following, the discrete-time dynamics of Σ is supposed to be affine and given by

$$\tilde{\Sigma}_{k+1} = A_k^d \Sigma_k + F_k^d + B_k^d \lambda, \tag{7}$$

where $\lambda := (\lambda_1 \lambda_2)^T$, with $\lambda_1 \in -\mathrm{Sgn}(\sigma_{k+1})$ and $\lambda_2 \in -\mathrm{Sgn}(\dot{\sigma}_{k+1})$. At each time instant t_k, we have $\Sigma_k = \Sigma(t_k)$ but $\tilde{\Sigma}_{k+1}$ is in general not equal to $\Sigma(t_{k+1})$. If the dynamics (1) is LTI and exact, the discrete-time dynamics obtained using a ZOH discretization is exact and therefore $\tilde{\Sigma}_{k+1} = \Sigma(t_{k+1})$. The control input value at time t_k is computed as

$$u_k = G(1\beta)\lambda,$$

and therefore requires the value of λ, which is obtained as the solution of the following generalized equation

$$\begin{cases} \tilde{\Sigma}_{k+1} = A_k^d \Sigma_k + F_k^d + B_k^d \lambda \\ \lambda \in -\mathrm{Sgn}(\tilde{\Sigma}_{k+1}) \end{cases} \tag{8}$$

with unknowns λ and $\tilde{\Sigma}_{k+1}$.

2.3. The implicit twisting as a generalized equation

Let us analyze this system using tools from convex analysis and variational inequalities theory. First we introduce the normal cone, denoted by $\mathcal{N}_K(z)$, to a non-empty, closed convex set K at a point $z \in K$, and defined by $\mathcal{N}_K(z) = \{x \in \mathbb{R}^n \mid \langle x, y - z\rangle \le 0\ \forall\, y \in K\}$. The equivalence $\lambda \in -\mathrm{Sgn}(\tilde{\Sigma}_{k+1}) \Longleftrightarrow \tilde{\Sigma}_{k+1} \in -\mathcal{N}_{[-1,1]^2}(\lambda)$ with $[-1, 1]^2 = [-1, 1] \times [-1, 1]$, enables us to transform (8) into the generalized equation