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Water hammer mitigation via PDE-constrained optimization ☆

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ABSTRACT

This paper considers an optimal boundary control problem for fluid pipelines with terminal valve control. The goal is to minimize pressure fluctuation during valve closure, thus mitigating water hammer effects. We model the fluid flow by two coupled hyperbolic PDEs with given initial conditions and a boundary control governing valve actuation. To solve the optimal boundary control problem, we apply the control parameterization method to approximate the time-varying boundary control by a linear combination of basis functions, each of which depends on a set of decision parameters. Then, by using variational principles, we derive formulas for the gradient of the objective function (which measures pressure fluctuation) with respect to the decision parameters. Based on the gradient formulas obtained, we propose a gradient-based optimization method for solving the optimal boundary control problem. Numerical results demonstrate the capability of optimal boundary control to significantly reduce pressure fluctuation.

1. Introduction

When gases and liquids are transported over long distances through networked pipelines, flow impulses and periodic excitations often induce unwanted transient dynamics. Such transient dynamics can adversely affect working performance, and can even destroy key components in the pipeline network, through the generation of fluid–structure interactive vibration and noise. Water hammer, also known as hydraulic shock, is one of the most common transient dynamics in pipelines. It is caused by sudden changes in the motion state, such as a complete halt or a reversal of flow direction. The pressure wave caused by water hammer is the main reason for pipeline noise and vibration. Mitigation strategies for water hammer are numerous and here we refer to just a few, such as those for oil pipelines (Xu, Dong, Ren, Jiang, & Yu, 2015), air compressor pipelines (Lee & Ngoh, 2002), spacecraft propulsion systems (Lecourt & Steelant, 2007), heat exchange systems in nuclear reactors (Erath, Nowotny, & Maetz, 1999; Tian, Su, Wang, Qiu, & Xiao, 2008) and even cardiovascular flow in human blood vessels (Pedley, 1980).

This paper models water hammer mitigation by an optimal boundary control problem governed by hyperbolic PDEs (Chen, Ren, Xu, & Loxton, 2015). We consider the benchmark pipeline system shown in Fig. 1, where a pipeline of length L is used to transport fluid from a reservoir to a terminus. In the literature, fluid flow is typically modeled using the well-known Navier–Stokes equations; related control studies include mixing, stabilization, and optimal shape design (Aamo & Krstic, 2002; Balogh, Aamo, & Krstic, 2005). For pipelines, the simplified version of the full Navier–Stokes model is commonly used to analyze and mitigate water hammer phenomena. This simplified model, known as the pipeline transmission PDE model, is defined by the following nonlinear hyperbolic PDE system (Ghidaoui, 2004; Wylie, Streeter, & Suo, 1993):

$$\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial l} + \frac{f}{2D}v|v| = 0, \tag{1a}$$

$$\frac{\partial p}{\partial t} + \rho c^2\frac{\partial v}{\partial l} = 0, \tag{1b}$$

where $l \in [0, L]$ is the spatial variable, $t \in [0, T]$ is the time variable, $v = v(l, t)$ is the flow velocity, $p = p(l, t)$ is the pressure drop, D is the diameter of the pipeline, c is the wave velocity, f is the Darcy–Weisbach friction factor, and ρ is the flow density. This model is also widely used to simulate hydraulic dynamics in irrigation

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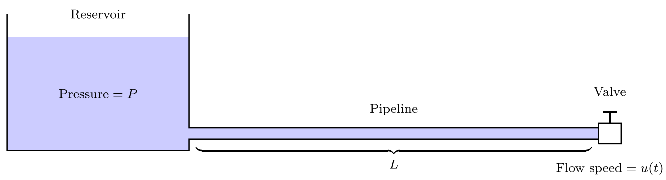

Fig. 1. General layout of the pipeline system.

canals (Litrico, Fromion, Baume, Arranja, & Rijo, 2005; Mareels et al., 2005; Ooi, Krutzen, & Weyer, 2005).

The initial conditions for system (1) are

$$p(l,0)=\bar{p}_0(l), \quad v(l,0)=\bar{v}_0(l), \quad l\in[0,L], \tag{2}$$

where $\bar{p}_0(l)$ and $\bar{v}_0(l)$ are given functions describing the initial pressure and velocity profiles respectively. Moreover, the boundary conditions are given by

$$p(0,t)=P, \quad v(L,t)=u(t), \quad t\in[0,T], \tag{3}$$

where P is the pressure generated by the reservoir, and $u(t)$ is a boundary control function. System (1)–(3) is called the state system. Note that $u(t)=0$ corresponds to a closed valve (zero flow velocity), and $u(t)=u_{\max}$ corresponds to a completely open valve (maximum flow velocity). Since the valve is initially fully open,

$$u(0)=u_{\max}. \tag{4}$$

Moreover, since the valve is required to be closed at the terminal time $t=T$, we impose the following terminal constraint:

$$u(T)=0. \tag{5}$$

Finally, to ensure that the valve is not re-opened during the time horizon, we have the following derivative constraint:

$$\dot{u}(t)\le 0, \quad t\in[0,T]. \tag{6}$$

Shutting off the valve suddenly will cause an oscillating pressure wave (i.e., water hammer) to propagate through the pipeline at high speed (Bergnt, Simpson, & Sijamhodzic, 1991). This pressure fluctuation must be controlled to avoid serious pipeline damage (Asli, Naghiyev, & Haghi, 2010; Schmitt, Pluvinage, Hadj-Taieb, & Akid, 2006). Thus, for the pipeline system shown in Fig. 1, our goal is to choose a continuous boundary control $u(t)$, in accordance with constraints (4)–(6), to minimize the following objective function as proposed in Atanov, Evseeva, and Work (1998) for open channel flows:

$$J(u)=\frac{1}{T}\int_0^T\left[\frac{p(L,t)-\hat{p}(L)}{\bar{P}}\right]^{2\gamma}dt+\frac{1}{LT}\int_0^T\int_0^L\left[\frac{p(l,t)-\hat{p}(l)}{\bar{P}}\right]^{2\gamma}dl\,dt, \tag{7}$$

where γ is a positive integer, $\bar{P}$ is a given constant, $\hat{p}(l)$ is a given function expressing the target pressure profile along the pipeline and $p(l,t)$ is the solution of the state system (1)–(3). Our optimal boundary control problem is thus stated as: Given the system (1) with initial conditions (2) and boundary conditions (3), choose the boundary control $u(t)$ to minimize the objective function (7) subject to the initial condition (4), the terminal control constraint (5) and the derivative constraint (6). This problem is referred to as Problem P_0.

In Chen et al. (2015), we developed a discretize-then-optimize computational approach for solving Problem P_0. This approach involves first using the finite-difference method to approximate the PDE model (1)–(3) by a system of ODEs, then applying control parameterization (Teo, Goh, & Wong, 1991) to approximate the boundary control by a piecewise-linear or piecewise-quadratic function. We call this approach the CP-ODE approach, as it involves using control parameterization to solve a conventional ODE optimal control problem, which is obtained from the original PDE problem via the finite-difference method.

In this paper, we propose an alternative computational approach in which control parameterization is applied directly to the original PDE model. We refer to this new approach as the CP-PDE approach. The advantage of CP-PDE over CP-ODE is that one layer of approximation is removed: Problem P_0 is solved directly using control parameterization; there is no need to first approximate it by a conventional ODE optimal control problem. Both CP-PDE and

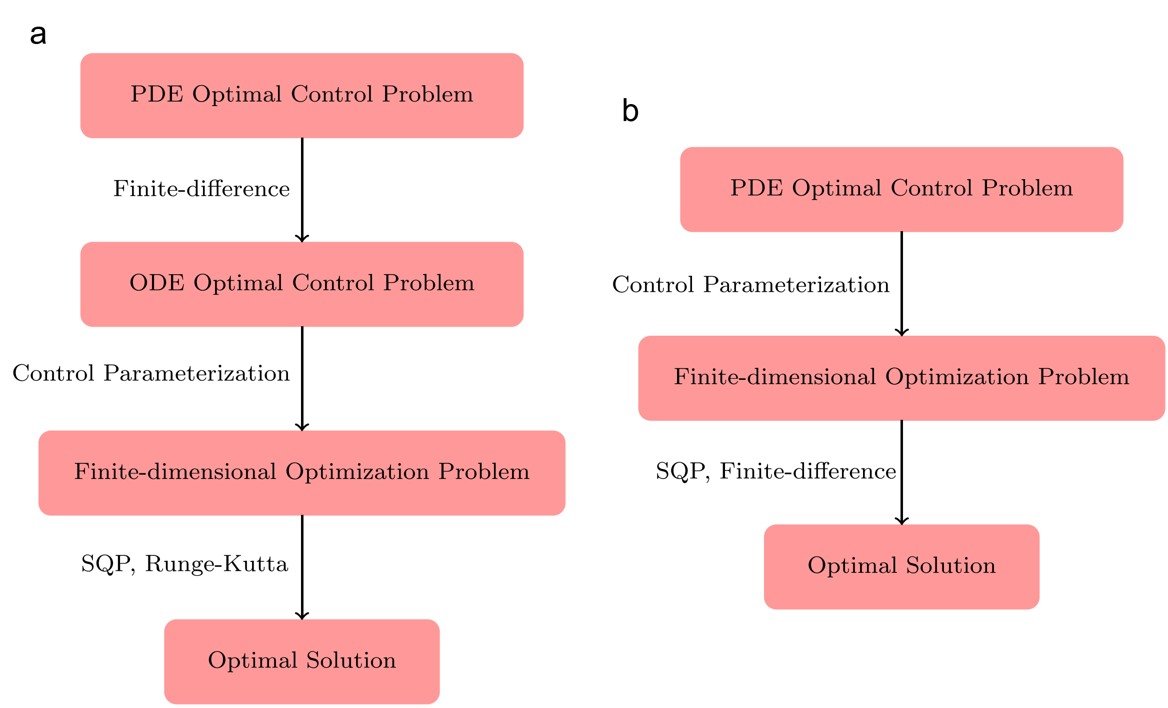

Fig. 2. Comparing the (a) existing approach in Chen et al. (2015) with the (b) new approach described in this paper.