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Linear interpolation based controller design for trajectory tracking under uncertainties: Application to mobile robots

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ABSTRACT

The problem of trajectory tracking control in mobile robots under uncertainties is addressed in this paper. Following the results of mobile robots trajectory tracking reported in (Scaglia et al., 2010), the problem of model errors is focused and the zero convergence of tracking errors under polynomial uncertainties is demonstrated. A simple scheme is obtained, which can be easily implemented. Simulation and experimental results are presented and discussed, showing the good performance of the controller.

1. Introduction

This paper addresses the problem of trajectory tracking control in mobile robots under uncertainties. The use of trajectory tracking for mobile robots is justified in structured working spaces as well as in partially structured workspaces, where unexpected obstacles can be found during the navigation. In the first case, the reference trajectory can be set from a global trajectory planner. In the second case, the algorithms used to avoid obstacles usually re-plan the trajectory in order to avoid a collision generating a new reference trajectory from this point on. In general, the objective is to find the control actions that make the mobile robot to reach a Cartesian position (x_{ref}, y_{ref}) with a pre-established orientation θ at each sampling period. These combined actions result in tracking the desired trajectory of the mobile robot.

Several tracking controllers designed by using a linearized model have been reported. In Oriolo, Luca & Vendittelli (2002), a dynamic feedback linearization technique is used to control a mobile robot platform. Usually, the desired trajectory is obtained by using a reference virtual robot; therefore, all the kinematic constraints are implicitly considered by the desired trajectory. The control inputs are mostly obtained by a combination of feedforward inputs, calculated from the desired trajectory, and the feedback control law. In Klancar & Skrjanc (2007), a model-predictive trajectory-tracking control applied to a mobile robot is presented. Linearized tracking-error dynamics is used to predict the system future behavior and a control law is derived from a quadratic cost function penalizing the system tracking error and the control effort. In Blažič (2011), a kinematic model is proposed where the transformation between the robot posture and the system state is bijective. The authors show that the global asymptotic stability of the system is achieved if the reference velocities satisfy the condition of persistent excitation.

In Scaglia, Quintero, Mut & Di Sciascio (2009), a novel trajectory-tracking controller has been presented. The originality of this control approach is based on the application of linear algebra for trajectory tracking, where the control actions are obtained by solving a system of linear equations. The methodology developed for tracking the desired trajectory (x_{ref} , y_{ref}) is based on determining the trajectories of the remaining state variables, thus the tracking error tends to zero. These states are determined by analyzing the conditions so that the system of linear equations has exact solution. In order to achieve this objective, only two control variables are available: the linear velocity (V) and the angular velocity (W) of the robot, Fig. 1. This design technique has been applied successfully in several systems (Scaglia et al., 2009; Rosales, Scaglia, Mut & Di Sciascio, 2011; Rosales, Scaglia, Mut & Di Sciascio, 2009; Serrano, Scaglia, Godoy, Mut & Ortiz, 2013).

The authors in Scaglia, Rosales, Quintero, Mut & Agarwal (2010), present a novel linear interpolation based methodology to design control algorithms for trajectory tracking of mobile vehicles. In that work, it is assumed that the evolution of the system

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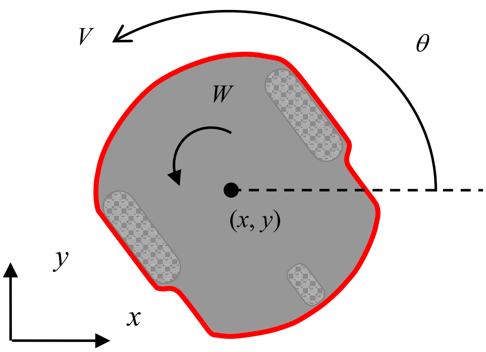

Fig. 1. Geometric description of the mobile robot.

can be approximated by a linear interpolation at each sampling time. A novel approach for trajectory tracking of a mobile-robots formation by using linear algebra theory and numerical methods is presented in Rosales et al. (2011). Using this strategy, the formation of mobile robots is able to change its configuration (shape and size) and follow different trajectories in a precise way, minimizing the tracking and formation errors. To face up to sudden velocity changes and to improve the performance of the system, Rosales et al. (2009) proposed working with the dynamic model of the mobile robot.

This paper provides a positive answer to the challenging problem of designing controllers for trajectory tracking in multivariable nonlinear systems with additive uncertainty. This problem has attracted the interest of many researchers and several solutions are reported in the literature (see for instance, Lee, Lin, Lim & Lee (2009), Farooq, Hasan, Hanif, Amer & Asad (2014), Wang (2012), Scaglia et al. (2009), and many others). The proposed controllers are rather complicated and require high computation cost.

The controller is derived by using the discrete state space model equations. This simple approach suggests that knowing the value of the desired state, a value for the control action forcing the system to move from its current state to the desired one can be computed. A system of linear equations needs to be solved to compute the control actions to carry the current outputs to the desired values. The main contribution of this work is the extension of the proposed methodology, based upon easily understandable concepts and without requiring complex calculations to attain the control signal, to design the tracking control of a mobile with uncertainties in the model. The methodology proposed here yields trajectory tracking controllers with low computational cost, small tracking error and low control effort. Additionally, our controller shows to be robust under disturbances in the control actions. Due to its mathematical formulation, our approach can also be implemented embedded (it does not compute higher order derivatives, exponentiation or complex trigonometric functions). Another contribution of this paper is the application of Monte Carlo (MC) based sampling experiment in the simulations. The controller parameters can be computed to minimize a cost index, here being determined by the Monte Carlo (MC) experiment, and the theoretical results are validated by simulations and experimentation.

It is noteworthy that due to the above mentioned characteristics, the computing power required to perform the mathematical operations is low. Thence it is possible to implement the algorithm in any controller with low computing capacity. Furthermore, the developed algorithm is easier to implement in a real system because the use of discrete equations allows direct adaptation to any computer system or programmable device running sequential instructions at a programmable clock speed. Thus, other than the simplicity of the controller, one great advantage of this approach is the use of discrete-time equations, simplifying its implementation on a computer system. The proof of the zero-convergence of the tracking error under uncertainty is another main contribution of this work.

The paper is organized as follows: in the next sections, some results from previous works are summarized to make this paper self-contained. In Section 2, the kinematical model of the mobile robot is presented, and the controller design methodology is shown in Section 3. In Section 4, the controller parameters are tuned by the MC experiment and theoretical results are validated with simulation results of the control algorithm. Three experimental results using a mobile robot Pioneer 3AT are presented in Section 5. Finally, the conclusions and some topics that will be addressed in future contributions are outlined in Section 6.

2. Kinematic model of the mobile robot and controller design

2.1. Kinematic model of the mobile robot

A simple nonlinear kinematic model for a mobile robot shown in Fig. 1, represented by (1), will be used

$$\begin{cases} \dot{x} = V\cos\theta \\ \dot{y} = V\sin\theta \\ \dot{\theta} = W \end{cases} \tag{1}$$

where, V: linear velocity of the mobile robot, W: angular velocity of the mobile robot, (x, y): Cartesian position, θ: mobile robot orientation. This model has been used in several recent papers such as Blažič (2011), Rosales et al., (2011), Rossomando, Soria & Carelli (2011) and Resende, Carelli & Sarcinelli-Filho (2013). Note that the dynamics of the mobile as well as those from the actuators are not initially considered. They will be taken into account later on as uncertainties in the model (1).

2.2. Controller design

The goal is to find the values of V and W so that the mobile robot may follow a pre-established trajectory (x_{ref}, y_{ref}) with a minimum error. The values of $x(t)$, $y(t)$, $\theta(t)$, $V(t)$ and $W(t)$ at discrete time $t = nT_s$, where T_s is the sampling time, and $n \in \{0, 1, 2, \cdots\}$ will be denoted as x_n, y_n, θ_n, V_n and W_n, respectively.

From (1), it follows,

$$\begin{cases} x_{n+1} = x_n + \int_{nT_s}^{(n+1)T_s} V\cos\theta dt \\ y_{n+1} = y_n + \int_{nT_s}^{(n+1)T_s} V\sin\theta dt \\ \theta_{n+1} = \theta_n + \int_{nT_s}^{(n+1)T_s} W dt \end{cases} \tag{2}$$

where the control actions V and W remain constant in the interval $nT_s \le t < (n+1)T_s$ and equal to V_n and W_n. Then, for $nT_s \le t < (n+1)T_s$,

$$\begin{cases} \theta(t) = \theta_n + W_n(t - nT_s) \\ \theta_{n+1} = \theta_n + W_nT_s \end{cases} \tag{3}$$

Then, from (2) and (3), it follows

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ \theta_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \\ \theta_n \end{bmatrix} + \begin{bmatrix} \frac{1}{W_n}\{\sin\theta_{n+1} - \sin\theta_n\} & 0 \\ \frac{1}{W_n}\{\cos\theta_n - \cos\theta_{n+1}\} & 0 \\ 0 & T_s \end{bmatrix} \begin{bmatrix} V_n \\ W_n \end{bmatrix} \tag{4}$$

Denote by $\mathbf{z}_n$ the state vector and $\Delta\mathbf{z}_n$ its increment, that is