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Stable recursive canonical variate state space modeling for time-varying processes

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ABSTRACT

An adaptive recursive process modeling approach is developed to improve the accuracy of modeling time-varying processes. We adopt the exponential weighted moving average approach to update the covariance and cross-covariance of past and future observation vectors. Forgetting factors are adjusted in the recursive modeling process based on the residual of model outputs. To ensure the stability of the identified model, we introduce a constrained nonlinear optimization approach and propose a stable recursive canonical variate state space modeling (SRCVSS) method. The performance of the proposed method is illustrated with an open-loop numerical example and simulation with the closed-loop data from a continuous stirred tank heater (CSTH) system. The results indicate that the accuracy of proposed SRCVSS modeling method is higher than that of state space modeling with traditional canonical variate analysis.

1. Introduction

For the past decades, subspace identification methods have attracted interest for process modeling, monitoring and control. The conventional subspace identification methods include canonical variate analysis (CVA), Numerical algorithms for Subspace State Space System IDentification (N4SID), and Multi-variable Output-Error State sPace (MOESP) (Qin, 2006). According to the unifying theory proposed by Van Overschee and De Moor (1996), these algorithms can be interpreted as singular value decomposition of a weighted matrix. Juricek, Seborg, and Larimore (2005) demonstrated that subspace models based on CVA and N4SID outperformed regression models based on partial least squares (PLS) and constrained categorical regression. They also demonstrated that the CVA model was more accurate than N4SID. Negiz and Cinar (1997) compared the application of PLS regression, balanced realization and canonical variate state space (CVSS) modeling technique in identifying stationary vector autoregressive moving average type of time series models in state space. They reported that balanced realization and PLS do not provide optimal state variables that are orthogonal, but orthogonal states' PLS and CVSS realization give minimal state variables that are orthogonal.

CVA was first developed by Larimore (1990). There are many successful examples for applying the CVA approach. Pilgram, Judd, and Mees (2002) used CVA for the description of random processes. Akaike (1998) applied the concept of canonical correlation analysis to stochastic realization theory and system identification. Larimore and Baillieul (1990) described a CVA approach for state space identification and filtering of linear and nonlinear systems. Negiz and Cinar (1997a) applied CVSS model to multivariable statistical monitoring and illustrated its performance in milk pasteurization process. Stubbs, Zhang, and Morris (2012) applied CVA to fault detection and diagnosis and used the Tennessee Eastman (TE) process simulator for case studies. Juricek, Seborg, and Larimore (2004) applied CVA to modeling a nonlinear continuous stirred tank reactor. CVA shows good modeling performance for linear processes and stationary operating conditions, but many processes are time-varying and often face changes in process operating conditions. Thus, it is necessary to develop a recursive method for model adaptation to changes in the mode of operation of time-varying processes.

Mercère, Bako, and Lecœuche (2008) developed a recursive formulation of the MOESP identification approach. Mercère and Lovera (2007) proposed online implementation of the MOESP methods based on instrumental variable versions of the propagator technique for signal subspace estimation. Lovera, Gustafsson, and Verhaegen (2000) proposed several recursive formulations for the MOESP algorithms. Wu, Yang, and Song (2008) proposed a recursive subspace identification method for predicting time-varying stochastic systems,

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which is to estimate the Toeplitz matrices by using the Vector Auto Regressive with eXogenous input (VARX) models. Kameyama and Ohsumi (2005) proposed a subspace method for predicting time-invariant or varying stochastic systems in the 4SID framework. Pavel and Vladimir (2006) proposed a different online recursive identification approach based on the interpretation of 4SID methods in the least squares sense. Houtzager, van Wingerden, and Verhaegen (2012) presented a recursive identification approach with constant forgetting factor based on predictor-based subspace methods in either open-loop or in closed-loop. Lee and Lee (2008) proposed a state space model based on CVA using mean, covariance, and correlation updating with variable forgetting factor based on the norm between two consecutive measurements. Although the variable forgetting factor was adopted, we still do not know how much the values should be changed to reduce the residual between the model outputs and measurements. To our best knowledge, no constraint conditions are used in the methods mentioned above to ensure the stability of the recursive model.

Measurement noise, plant-model mismatch and nonstationary disturbances acting on a time-varying and/or nonlinear process may cause a recursive model to become unstable even though the process is stable. The coefficient matrices of state space model need to be updated with every new measurements set under the condition that the identified model is stable. In this paper, we focus on adaptive system identification based on canonical variates and introduce a nonlinear constrained optimization method to ensure the stability of the recursive model. To obtain models that accurately reflect the current dynamics of the system and forecast well, we propose a new method for adjusting the forgetting factor based on the residual of model outputs. Then we propose an approach based on stable recursive canonical variate state space (SRCVSS) modeling and illustrate its performance with simulation of the continuous stirred tank heater (CSTH) system.

The remainder of the paper is organized as follows. Section 2 reviews the state space modeling based on conventional CVA. Section 3 presents the SRCVSS modeling technique, and proposes a new method to adjust the forgetting factor based on the residual between model outputs and real measurements. Section 4 presents a numerical example to illustrate the consistent estimation of eigenvalues of state matrix. Section 5 outlines the CSTH process simulator and illustrates the stability of the identified model and performance of SRCVSS method for time-varying processes. Simulation results compared with that of CVA are also given. Conclusions are presented in Section 6.

2. State space modeling based on canonical variate analysis

The multivariable state space model can be expressed as (Qin, 2006)

$$x_{(k+1)} = \mathbf{A}x_{(k)} + \mathbf{B}u_{(k)} + \mathbf{K}e_{(k)} \tag{1}$$

$$y_{(k)} = \mathbf{C}x_{(k)} + \mathbf{D}u_{(k)} + e_{(k)} \tag{2}$$

where $x_{(k)} \in \mathbb{R}^n$, $u_{(k)} \in \mathbb{R}^{m_u}$ and $y_{(k)} \in \mathbb{R}^{m_y}$ are state, input and observation vectors at sampling time k, $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times m_u}$, $\mathbf{C} \in \mathbb{R}^{m_y\times n}$, $\mathbf{D} \in \mathbb{R}^{m_y\times m_u}$ and $\mathbf{K} \in \mathbb{R}^n$ are state matrix, input matrix, output matrix, direct feed-through matrix and Kalman gain matrix, respectively. $e_{(k)}$ is stochastic disturbance, which is assumed to be zero-mean, normally distributed white noise.

An invertible linear transformation of the state does not change the input–output behavior of a state-space system. Therefore, we can only determine the system matrices up to a similarity transformation $\mathbf{T} \in \mathbb{R}^{n\times n}$, $\mathbf{T}^{-1}\mathbf{AT}$, $\mathbf{T}^{-1}\mathbf{B}$, $\mathbf{T}^{-1}\mathbf{K}$, $\mathbf{CT}$ and $\mathbf{D}$. The identification problem can be formulated as: given the input sequence u, output sequence y, find all system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, and $\mathbf{K}$, if they exist, up to a global similarity transformation recursively (Houtzager, van Wingerden, & Verhaegen, 2009b).

In the CVA approach, the measurement vector is expanded by past and future measurements, to form the past, extended past and future observation vectors $z_{p(r)}$, $z^+_{p(r)}$ and $y_{f(r)}$, respectively, where r denotes a generic index:

$$z_{p(r)} = \begin{bmatrix} y_{(r-1)} \\ u_{(r-1)} \\ y_{(r-2)} \\ u_{(r-2)} \\ \vdots \\ y_{(r-p)} \\ u_{(r-p)} \end{bmatrix} \in \mathbb{R}^{(m_u+m_y)\cdot p} \tag{3}$$

$$z^+_{p(r)} = \begin{bmatrix} y_{(r-1)} \\ u_{(r-1)} \\ y_{(r-2)} \\ u_{(r-2)} \\ \vdots \\ y_{(r-p-1)} \\ u_{(r-p-1)} \end{bmatrix} \in \mathbb{R}^{(m_u+m_y)\cdot (p+1)} \tag{4}$$

$$y_{f(r)} = \begin{bmatrix} y_{(r)} \\ y_{(r+1)} \\ \vdots \\ y_{(r+f-1)} \end{bmatrix} \in \mathbb{R}^{m_y\cdot f} \tag{5}$$

where p and f represent the length of the past and future observation windows, respectively. In general, $f \le p$ (Houtzager et al., 2009b, 2012).

Following the terminologies and equations in Odiowei and Cao (2010) and setting $r = p+1, p+2, \ldots, p+N$, the past and future Hankel matrices $\mathbf{Z}_p \in \mathbb{R}^{(m_u+m_y)p\times N}$ and $\mathbf{Y}_f \in \mathbb{R}^{m_y f\times N}$ are defined as

$$\mathbf{Z}_p = [z_{p(p+1)} \; z_{p(p+2)} \; \cdots \; z_{p(p+N)}] \tag{6}$$

$$\mathbf{Y}_f = [y_{f(f+1)} \; y_{f(f+2)} \; \cdots \; y_{f(f+N)}] \tag{7}$$

For a set of variables measured by l observations, the last two elements of $z_{p(p+1)}$ in Eq. (3) is $y_{(1)}, u_{(1)}$, whereas the last element of $y_{f(f+N)}$ in Eq. (5) would be $y_{(l)}$. Therefore, the maximum number of columns of these Hankel matrices is $N = l-p-f+1$.

The covariance and cross-covariance matrices of the past and future observations can be computed by using

$$\mathbf{\Sigma}_{pp} := \mathbf{Z}_p\mathbf{Z}_p^T/(N-1) \tag{8}$$

$$\mathbf{\Sigma}_{ff} := \mathbf{Y}_f\mathbf{Y}_f^T/(N-1) \tag{9}$$

$$\mathbf{\Sigma}_{fp} := \mathbf{Y}_f\mathbf{Z}_p^T/(N-1) \tag{10}$$

CVA finds the best linear combinations between $a^T y_{f(r)}$, $a \in \mathbb{R}^{m_y f}$ and $b^T z_{p(r)}$, $b \in \mathbb{R}^{(m_u+m_y)p}$ so that the correlation between the past and future observations is maximized (Odiowei & Cao, 2010):

$$\rho_{fp}(a,b) = \frac{a^T\mathbf{\Sigma}_{fp}b}{(a^T\mathbf{\Sigma}_{ff}a)^{1/2}(b^T\mathbf{\Sigma}_{pp}b)^{1/2}} \tag{11}$$

Define $u = \mathbf{\Sigma}_{ff}^{1/2}a$ and $v = \mathbf{\Sigma}_{pp}^{1/2}b$. The optimization to determine u and v can be represented as

$$\max_{u,v} \quad u^T(\mathbf{\Sigma}_{ff}^{-1/2}\mathbf{\Sigma}_{fp}\mathbf{\Sigma}_{pp}^{-1/2})v$$
$$\text{s.t.} \quad u^Tu = 1, \;\; v^Tv = 1 \tag{12}$$

The solution of the optimization problem can be obtained through the singular value decomposition (SVD) of the scaled