Control Engineering Practice 29 (2014) 50–60

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Nonlinear thermal system identification using fractional Volterra series☆

Asma Maachou[a], Rachid Malti[a,*], Pierre Melchior[a], Jean-Luc Battaglia[b], Alain Oustaloup[a], Bruno Hay[c]

[a] Université de Bordeaux, Institut Polytechnique de Bordeaux, IMS UMR 5218 CNRS, 351 cours de la Libération, Bât A4, 33405 Talence Cedex, France
[b] Université de Bordeaux, I2M UMR 5295 CNRS, Esplanade des Arts et Métiers, 33405 Talence Cedex, France
[c] LNE, Laboratoire National de métrologie et d'Essais, 29 avenue Roger Hennequin, 78197 Trappes Cedex, France

ARTICLE INFO

Article history:
Received 27 September 2013
Accepted 28 February 2014
Available online 6 May 2014

Keywords:
System identification
Fractional model
Fractional calculus
Nonlinear model
Volterra series
Large temperature variation

ABSTRACT

Linear fractional differentiation models have already proven their efficacy in modeling thermal diffusive phenomena for small temperature variations involving constant thermal parameters such as thermal diffusivity and thermal conductivity. However, for large temperature variations, encountered in plasma torch or in machining in severe conditions, the thermal parameters are no longer constant, but vary along with the temperature. In such a context, thermal diffusive phenomena can no longer be modeled by linear fractional models. In this paper, a new class of nonlinear fractional models based on the Volterra series is proposed for modeling such nonlinear diffusive phenomena. More specifically, Volterra series are extended to fractional derivatives, and fractional orthogonal generating functions are used as Volterra kernels. The linear coefficients are estimated along with nonlinear fractional parameters of the Volterra kernels by nonlinear programming techniques. The fractional Volterra series are first used to identify thermal diffusion in an iron sample with data generated using the finite element method and temperature variations up to 700 K. For that purpose, the thermal properties of the iron sample have been characterized. Then, the fractional Volterra series are used to identify the thermal diffusion with experimental data obtained by injecting a heat flux generated by a 200 W laser beam in the iron sample with temperature variations of 150 K. It is shown that the identified model is always more accurate than the finite element model because it allows, in a single experiment, to take into account system uncertainties.

1. Introduction

This work is a part of a project which aims at developing a new device for heat flux estimation in tools during severe machining or in high enthalpy plasma flow. This estimation is based on temperature measurements near the heated area and on the inversion of a model which links the measured temperature to the heat flux. This paper focuses on the formulation of a reliable model consistent with large temperature variations leading to a nonlinear behavior due to temperature dependent thermal properties of the studied system.

Simulating a finite element model requires knowing all the thermal characteristics (specific heat, thermal conductivity, density) of the studied thermal system, and hence requires many prior experiments for thermal characterization. In spite of this knowledge, a finite element model is generally less accurate than an identified model, because it is difficult to take into account the following:

(i) the exact thermocouple location due to assemblage accuracy,
(ii) the exact geometry of the contact surface between the sensor and the sample,
(iii) boundary between the thermocouple and the sample which is never ideal because the thermocouple is usually glued to the material and there might be an air gap between both components,
(iv) the nonlinear dynamical behavior of the sensor.

The identified model includes all the system uncertainties mentioned above and may be obtained in a single experiment. It is thus expected to be more accurate.

☆A first preliminary paper was presented at the 4th IFAC Workshop on Fractional Differentiation and Its Applications, Badajoz, Spain, 2011 (Maachou et al., 2010), and a second one at the 18th World IFAC Congress, Milan, Italy, 2011 (Maachou et al., 2011). Here is a revisited and enhanced version of the two preliminary papers.

* Corresponding author.
E-mail addresses: asma.maachou@ims-bordeaux.fr (A. Maachou), rachid.malti@ims-bordeaux.fr (R. Malti), pierre.melchior@ims-bordeaux.fr (P. Melchior), jean-luc.battaglia@bordeaux.ensam.fr (J.-L. Battaglia), bruno.hay@lne.fr (B. Hay).

http://dx.doi.org/10.1016/j.conengprac.2014.02.023
0967-0661/

For short time experiments, thermal systems are regarded as semi-infinite media. Therefore, a fractional model appears naturally as an exact solution to express the temperature at a given point in the system versus the heat flux (Battaglia, 2008; Oldham & Spanier, 1972). Different system identification methods based on fractional linear models, developed in the literature, are summarized in the state of the art (Malti, Victor, & Oustaloup, 2008). For instance, Cois, Oustaloup, Battaglia, and Battaglia (2000) extended the output error to fractional models by using a modal representation. Cois, Oustaloup, Poinot, and Battaglia (2001) proposed to use a prediction error method combined to instrumental variable and state variable filters. This method was enhanced in Victor, Malti, Garnier, and Oustaloup (2013) by choosing optimal instrumental variables. Battaglia and Kusiak (2005) implemented recursive least squares for online parameters estimation. Malti, Aoun, Levron, and Oustaloup (2005), Aoun, Malti, Levron, and Oustaloup (2007), and Akçay (2008) used fractional orthogonal basis for system approximation. Malti, Raïssi, Thomassin, and Khemane (2010) and Khemane, Malti, Raïssi, and Moreau (2012) proposed a set-membership approach for system identification using fractional models using frequency-domain data. Fractional multi-models have been proposed for modeling gastrocnemius frog muscle in Sommacal et al. (2007, 2008).

For significant temperature variations, linear models are not accurate enough (Maachou et al., 2012; Gabano, Poinot, & Kanoun, 2010). Recently, Gabano et al. (2010) proposed the identification of an ARMCO (American Rolling Mill COmpany) iron sample using a fractional linear parameter varying (LPV) model. In a direct relationship with the results of this paper, a theoretical study has recently been carried out for determining analytical expressions of the Volterra series in Battaglia, Maachou, Malti, and Melchior (2013). In this paper, Volterra series are extended to fractional derivatives with kernels generated using fractional orthogonal generating functions. Two reasons motivate this choice. First, nonlinear model decomposition into Volterra series allows us to separate the contribution of the linear and the nonlinear parts. Secondly, the Volterra series may be seen as a generalization of linear models. Volterra series have already been used, in the literature, for the approximation and control of eddy current brake in Simeu and Georges (1996) or recently for modeling a greenhouse temperature in Gruber et al. (2011). The Hammerstein model based on Volterra series helped Angerer, Hintz, and Schröder (2004) identifying a nonlinear mechatronic system. Bibes (2004) modeled the process of water supplies using Volterra series developed on orthonormal basis functions. Casenave (2011) used implicit Volterra model form for the time-local formulation. This paper deals with the extension of Volterra series to fractional derivatives and its application in thermal modeling.

Heat transfer in a semi-infinite medium is modeled in Section 2. It is shown how linear fractional differentiation models are obtained by solving the heat equation under some initial and boundary conditions. Then, the limits of linear models are pointed out for large temperature variations which justifies the use of nonlinear fractional models. Then, in Section 3, a mathematical background is presented regarding fractional calculus and the Volterra series are presented as a generalization of linear models. In Section 4, a new class of models is presented as fractional Volterra series. Fractional generating orthogonal functions are used as Volterra kernels. In a system identification context, parameter estimation is developed in Section 5. Either linear coefficients of the Volterra series are estimated by least squares, or the linear coefficients are estimated along with nonlinear fractional parameters of the Volterra kernels by nonlinear programming techniques. In Section 6, the fractional Volterra series are used to model temperature variations versus heat flux for a monovariable system using two sets of data. This system is constituted of a simple thick iron disk, with one face subject to a uniform heat flux and with a thermocouple embedded close to the heated area. The first set of data is generated using the finite element method with the temperature dependent thermal properties of the iron sample characterized, for the aim of the simulation, by the *Laboratoire National de métrologie et d'Essais* (LNE), the French National Metrology Institute (NMI). The second set of data is obtained with an experimental set-up on the same thermal system. The fractional Volterra series are used to model temperature variations for the two sets of data. Moreover, the measured output, the finite element model output and the Volterra model output are compared to each other. It is shown that the identified Volterra model is more accurate than the finite element model because it allows, in a single experiment, to take into account system uncertainties (i)–(iv) described in Section 1. Finally in the conclusion, the main results of the paper are recalled.

2. Fractional calculus for thermal modeling: from small to large temperature variations

Fractional calculus has witnessed a growing interest in various fields as described in the *old* and the *recent history of fractional calculus* by Machado, Kiryakova, and Mainardi (2010, 2011).

On the basis of the heat equation characterizing the heat transfer in a semi-infinite medium (Fig. 1), Battaglia, Le Lay, Batsale, Oustaloup, and Cois (2000) showed that a fractional model is pertinent in modeling heat transfer at a distance x from the heated surface. The diffusion phenomenon is governed by the linear heat diffusion equation

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2} \quad \text{for } 0 < x < \infty, \ \ t > 0 \tag{1}$$

where α is the thermal diffusivity. For convenience, isothermal and null states are chosen

$$T(x,t) = 0, \quad \text{for } 0 \le x < \infty, \ \text{at } t = 0 \tag{2}$$

The boundary conditions are

$$\begin{cases} -\lambda \dfrac{\partial T(x,t)}{\partial x} = \varphi(t) & \text{for } x = 0, \ \text{at } t > 0 \\ T(x,t) = 0 & \text{for } x \to \infty, \ \text{at } t > 0. \end{cases} \tag{3}$$

where λ is the thermal conductivity of a material and $\varphi(t)$ is the heat flux applied on its surface. Knowing that

$$\alpha = \frac{\lambda}{\rho C_p}, \tag{4}$$

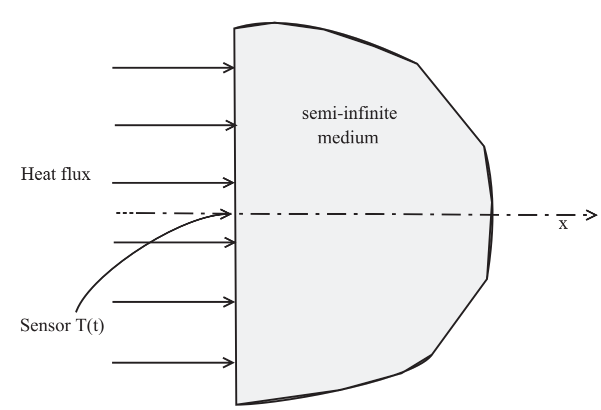

Fig. 1. Heat flux applied on a surface S of a semi-finite medium.