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Energy shaping control of an inverted flexible pendulum fixed to a cart

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ABSTRACT

Control of compliant mechanical systems is increasingly being researched for several applications including flexible link robots and ultra-precision positioning systems. The control problem in these systems is challenging, especially with gravity coupling and large deformations, because of inherent underactuation and the combination of lumped and distributed parameters of a nonlinear system. In this paper we consider an ultra-flexible inverted pendulum on a cart and propose a new nonlinear energy shaping controller to keep the pendulum at the upward position with the cart stopped at a desired location. The design is based on a model, obtained via the constrained Lagrange formulation, which previously has been validated experimentally. The controller design consists of a partial feedback linearization step followed by a standard PID controller acting on two passive outputs. Boundedness of all signals and (local) asymptotic stability of the desired equilibrium is theoretically established. Simulations and experimental evidence assess the performance of the proposed controller.

1. Introduction

The problem of stabilization of underactuated mechanical systems, both in the domain of ordinary and partial differential equations, has been widely addressed by several control researchers in recent years. In the domain of flexible mechanisms and robots, flexibility in the links is the main source of underactuation. If the deformations due to flexibility are small it is possible to use an *unconstrained* Lagrange formulation and invoke the Assumed Modes Method (AMM) (Meirovitch, 1975) to obtain a simple, finite-dimensional model—see Dwivedy and Eberhard (2006) for a recent literature review. This modeling procedure, however, is inapplicable for systems with large deformations, for which a *constrained* Euler–Lagrange (EL) formulation is required. This approach has been adopted in Patil and Gandhi (2014) to derive an accurate model for a single ultra-flexible link fixed to a cart. Potential energy change owing to ultra-large deformations in the presence of gravity is considered in Patil and Gandhi (2014) using the constant length of the beam as a holonomic constraint. For a survey on recent control techniques for this class of systems see Patil and Gandhi (2014), Torfs, Vuerinckx, Swevers, and Schoukens (1998), and Bayo (1987).

The objective of this paper is to design an energy shaping controller with *guaranteed stability properties* for the model of a single ultra-flexible link fixed to a cart reported in Patil and Gandhi (2014). As is well known (Ortega, Donaire, & Romero, 2016) the application of energy shaping controllers is stymied by the need to solve partial differential equations (PDEs) that identify the mechanical structure (Lagrangian or Hamiltonian) that is assigned to the closed-loop. To propose a truly constructive energy shaping scheme, that does not require the solution of PDEs, it was recently proposed in Donaire et al. (2016) to relax the constraint of preservation in closed-loop of the EL structure. The design in Donaire et al. (2016) proceeds in two steps, first, we apply a partial feedback linearization (PFL) (Spong, 1998) that transforms the system into Spong's normal form—if this system is still EL, two new passive outputs are immediately identified. Second, a classical PID around a suitable combination of these passive outputs completes the design.

It is shown in the paper that this technique, developed for standard EL systems in Donaire et al., is also applicable to the constrained EL system at hand. This extension is far from obvious, because the (lower order) dynamics that results from the projection of the system on the manifold defined by the constraint *is not* an EL system. In spite of this fact it is shown that because of the workless nature of the forces introduced by the constraints, it is still possible to identify the two new passive outputs to which the PID is applied.

The remainder of the paper is organized as follows. Section 2 presents the full constrained EL dynamics of the system and its

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reduced order projection. Section 3 presents the proposed energy shaping control algorithm. Section 4 presents the simulation results, while in Section 5 we show the experimental ones. Section 6 summarizes the work and outlines some future research.

Notation: Unless indicated otherwise, all vectors in the paper are column vectors. Given $n \in \mathbb{N}$, $e_i \in \mathbb{R}^n$ is the ith Euclidean basis vector of $\mathbb{R}^n$. For $x \in \mathbb{R}^n$, we denote $|x|^2 := x^\top x$. To simplify the expressions, the arguments of all mappings—that are assumed smooth—will be explicitly written only the first time that the mapping is defined. For a scalar function $V: \mathbb{R}^n \to \mathbb{R}$, we define $\nabla_x V := \left(\frac{\partial V}{\partial x}\right)^\top$ and $\nabla_x^2 V := \frac{\partial^2 V}{\partial x^2}$—when clear from the context the subindex in ∇ will be omitted.

2. System dynamics and problem formulation

In Patil and Gandhi (2014) a dynamic model that accurately describes the behavior of the single ultra-flexible link fixed to a cart depicted in Fig. 1 is reported. The main feature of this model, which distinguishes it from other models, is that to take into account large deformations of the link where its length is assumed *constant*—giving rise to a holonomic constraint. The model is rigorously developed using a constrained EL formalism, combined with a standard application of the AMM, and its validity is experimentally validated. In this section we present this model, first, in its constrained EL form and then in a reduced form—obtained via the elimination of the constrained equations.

2.1. Constrained Euler–Lagrange model

The model reported in Patil and Gandhi (2014) admits a constrained EL representation of the form

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + B(q) + R\dot{q} = e_3\tau + \lambda A(q),$$
$$\Gamma(q) = 0, \tag{1}$$

where $q = \text{col}(\theta, x_e, z) \in \mathbb{D} \times \mathbb{R}_+ \times \mathbb{R}$ are the generalized coordinates, $R \geq 0$ is a matrix of damping coefficients. $D > 0$ is the inertia matrix, $C\dot{q}$ are the Coriolis and centrifugal forces, B is a conservative force vector due to potential energy, τ is the control

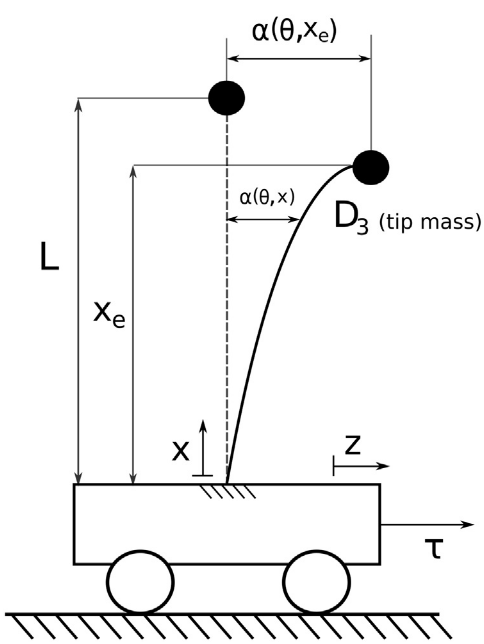

Fig. 1. Single ultra-flexible link with base excitation.

vector, λA is a vector of virtual forces due to the holonomic constraint, with λ being the Lagrange multiplier, and Γ is the (constant length) constraint function given by

$$\Gamma(q) := \int_0^{x_e} \sqrt{1 + \left[\theta\phi'(x)\right]^2}\, dx - L, \tag{2}$$

with $L > 0$ being the length of the link and ϕ being the mode shape function of the AMM (Meirovitch, 1975) reported in Laura, Pombo, and Susemihl (1974), that is,

$$\phi(x) = \cosh\left(\frac{\eta x}{L}\right) - \cos\left(\frac{\eta x}{L}\right) + \gamma\left[\sin\left(\frac{\eta x}{L}\right) - \sinh\left(\frac{\eta x}{L}\right)\right], \tag{3}$$

where η and γ are given in Table 1. The analysis made in Patil and Gandhi (2014) considers only one mode where the deflection $\alpha(\theta, x)$ is given by

$$\alpha(x, \theta) = \phi(x)\theta.$$

The different terms entering into (1) are defined as

$$D(q) := \begin{bmatrix} D_1(x_e) & 0 & D_2(x_e) \\ 0 & D_3 & 0 \\ D_2(x_e) & 0 & D_4 \end{bmatrix},$$

$$A(q) = \begin{bmatrix} A_1(\theta, x_e) \\ A_2(\theta, x_e) \\ 0 \end{bmatrix} := \nabla\Gamma(q), \quad R := \text{diag}\{R_1, 0, R_3\},$$

$$C(q, \dot{q}) := \begin{bmatrix} \frac{1}{2}C_1(x_e)\dot{x}_e & \delta(x_e, \dot{\theta}, \dot{z}) & \frac{1}{2}C_2(x_e)\dot{x}_e \\ -\delta(x_e, \dot{\theta}, \dot{z}) & 0 & -\frac{1}{2}C_2(x_e)\dot{\theta} \\ \frac{1}{2}C_2(x_e)\dot{x}_e & \frac{1}{2}C_2(x_e)\dot{\theta} & 0 \end{bmatrix},$$

with

$$\delta(x_e, \dot{\theta}, \dot{z}) := \frac{1}{2}C_1(x_e)\dot{\theta} + \frac{1}{2}C_2(x_e)\dot{z},$$

and

$$B(q) = \begin{bmatrix} B_1(\theta, x_e) \\ B_2(\theta, x_e) \\ 0 \end{bmatrix} := \nabla V(q) \tag{4}$$

where V is the potential energy of the system given by

Table 1
System parameters.

Parameter	Symbol	Value	Units
Pendulum cross section area	$\mathcal{A}_o$	8×10^{-6}	m^2
Young's modulus	E	9×10^{10}	$\frac{N}{m^2}$
Gravitational acceleration	g	9.81	$\frac{m}{seg^2}$
Moment of inertia	I	1.066×10^{-13}	$kg\,m^2$
Pendulum length	L	0.305	m
Tip mass	M	2.75×10^{-2}	kg
Cart mass	M_c	0.1	kg
Function of the system natural frequency	η	1.1741	–
Dimensionless constant	γ	0.9049	–
Pendulum density	ρ	8400	$\frac{kg}{m^3}$
Viscous friction at the pendulum base	R_1	9.86×10^{-4}	$\frac{kg}{seg}$
Viscous friction between the rail and the cart	R_3	7.69	$\frac{kg}{seg}$