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Multivariable system stabilization via discrete variable structure control

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A B S T R A C T

This paper presents the design of an incremental discrete variable structure controller, based on the minimization of the generalized minimum variance approach, to deal with multivariable plants and to handle different time delay of control loops between the input–output pairs. Additionally, a static precompensator, to decouple the control loops in multivariable systems, is developed not only to keep a good closed-loop behavior but also to decrease the control algorithm complexity. Numerical simulation examples are shown to illustrate the dynamic performance and the closed-loop stability for three multivariable systems.

1. Introduction

Since the pioneer work of Åström and Wittenmark (1973) that developed the minimum variance regulator, the Generalized Minimum Variance (GMV) controller for Multi-Input/Multi-Output (MIMO) systems has received a lot of interest in practical experiments and with many successful applications in the industry (Coelho, Gomes, Amaral, & Yamakami, 1990; Corradini & Orlando, 1997; Grimble & Majecki, 2013; Sbarbaro, Murray-Smith, & Valdes, 2004).

Several multivariable GMV controllers were derived under the assumption of an equal time delay for the input–output pairs. On one hand the *interactor* matrix is at the diagonal form. On the other hand, to deal with the problems associated with coupling and stability of the control loops for MIMO plants, many GMV control algorithms with non-diagonal *interactor* matrix were developed (Coelho et al., 1990; Dugard, Goodwin, & Xianya, 1984; Huang, Shah, & Kwok, 1997; Majecki & Grimble, 2004; Sendjaja & Kariwala, 2012; Wu, Du, & Qian, 2012).

The discrete version of the variable structure control (VSC) based on the input–output transfer function has been received a notable attention in the control research community worldwide (Corradini & Orlando, 1995; Furuta, 1993; Hamano & Kim, 2001, Chap. 8; Yu, Wang, & Li, 2012). In particular, when the variable structure control has been implemented based on the generalized minimum variance control, the literature shows different control algorithms to deal with parametric uncertainties and to give good dynamic for the controlled system. In addition, Corradini and Orlando (1997) have proposed a multivariable VSC based on the preliminary version of the VSC by using the monovariable minimum variance control conception and no comments for the dynamic coupling (*interactor* matrix) was presented. In particular, the VSC algorithm was evaluated in an underwater remotely operated vehicle that presents diagonal *interactor* matrix with unitary time delay and some synchronous setpoint essays were applied to assess the closed-loop stability.

This paper presents in Section 2 the multivariable VSC/GMV controller synthesis. Some control design characteristics are (i) VSC/GMV control algorithm of Corradini and Orlando (1997) is extended to include the non-diagonal *interactor* matrix; (ii) non-minimum phase processes can be assessed as shown in Coelho and do Amaral (1998) (the alone GMV controller has a serious problem to deal with this kind of process as shown in Clarke & Gawthrop (1975); (iii) incremental control conception is implemented to ensure setpoint tracking, and disturbance rejection (offset free).

Multivariable systems present distinct or multiple transport delay that often is difficult to control with monovariable control strategies. SISO control algorithms are appropriate if the coupling is low and the plant model is optimal. However, there are many multivariable processes in the industry with high coupling between the control loops and it is important to consider the coupling in the control system design. The synthesis of each control loop can be done individuality (Yamamoto & Shah, 1998). The basic idea is to decouple MIMO systems by dynamic or static precompensators, and deal with the decoupled system as a

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diagonal system, where monovariable controllers can be derived to the control loops. The implementation of a static precompensator for the GMV controller is shown in Section 3.

Finally, Section 4 shows the simulation results to the MIMO VSC/GMV controller with adequate use of the *interactor* matrix. In addition to this a precompensator design is also included to the GMV in order to eliminate the coupling between process variables when the system presents distinct delays between the input–output pairs (two different methodologies to deal with multivariable plants).

2. VSC/GMV multivariable control with *interactor* matrix

Assume that the process can be described by the following MIMO linear difference equation:

$$\mathbf{A}(q^{-1})\mathbf{y}(k) = \mathbf{B}(q^{-1})\mathbf{u}(k-1) \tag{1}$$

where $\mathbf{y}(k)$ and $\mathbf{u}(k) \in \Re^n$ are the process output and input vectors, respectively. The polynomial matrices $\mathbf{A}(q^{-1})$ and $\mathbf{B}(q^{-1}) \in \Re^{n\times n}$ have no common terms, are assumed to be known and are given by

$$\mathbf{A}(q^{-1}) = \mathbf{I} + \mathbf{A}_1 q^{-1} + \cdots + \mathbf{A}_{na} q^{-na} \tag{2}$$

$$\mathbf{B}(q^{-1}) = \mathbf{B}_o + \mathbf{B}_1 q^{-1} + \cdots + \mathbf{B}_{nb} q^{-nb} \tag{3}$$

where $\mathbf{I}$ is the identity matrix.

For discrete multivariable systems the *interactor* matrix, $\boldsymbol{\xi}(q)$, plays the same role as the time delay for SISO processes, with lower triangular form and satisfies the following equation (Huang et al., 1997):

$$\lim_{q^{-1}\to 0} q^{-1}\boldsymbol{\xi}(q)\mathbf{A}^{-1}(q^{-1})\mathbf{B}(q^{-1}) = \mathbf{K}, \quad \det(\mathbf{K}) > 0 \tag{4}$$

It is very important to include the *interactor* matrix for the GMV control design in order to compensate the coupling between the control loops and to guarantee good closed-loop stability and robustness. Thus, the proposed multivariable GMV controller differs from Corradini & Orlando (1997) design, by the presence not only the *interactor* matrix in the controlled system but also the implementation of an incremental control conception.

To obtain the GMV control, the signal $\mathbf{z_r}(k)$ is defined as

$$\mathbf{z_r}(k) = \boldsymbol{\xi}(q)\mathbf{y}_r(k-d) \tag{5}$$

where d is the maximum advance in $\boldsymbol{\xi}(q)$. The control objective is to minimize the variance of the controlled variable $\mathbf{s}(k+d)$ that produces a control signal that satisfies

$$\mathbf{s}(k+d) = \mathbf{T}(q^{-1})\boldsymbol{\xi}(q)\mathbf{e}(k) + \mathbf{P}(q^{-1})\Delta\mathbf{u}(k) \tag{6}$$

where $\mathbf{e}(k) = \mathbf{y}(k) - \mathbf{y}_r(k)$ is the tracking error vector for each output, $\Delta\mathbf{u}(k) = (1-q^{-1})\mathbf{u}(k)$ is the incremental control vector and the polynomial matrices $\mathbf{P}(q^{-1})$ and $\mathbf{T}(q^{-1})$ have the form

$$\mathbf{P}(q^{-1}) = \mathbf{P}_o + \mathbf{P}_1 q^{-1} + \cdots + \mathbf{P}_{np} q^{-np} \tag{7}$$

Table 1
Closed-loop dynamic.

$\Delta\mathbf{P}(q^{-1})$	The stability, Eq. (20), is guaranteed?	
$p_o(1-q^{-1})\mathbf{I}_2$	VSC_C	VSC_ξ
0.01	No	Yes
0.1	No	Yes
0.2	No	Yes
0.3	yes	yes
0.4	Yes	Yes

$$\mathbf{T}(q^{-1}) = \mathbf{T}_o + \mathbf{T}_1 q^{-1} + \cdots + \mathbf{T}_{nt} q^{-nt} \tag{8}$$

The incremental control vector, which satisfies Eq. (6), is calculated by

$$\mathbf{R}(q^{-1})\Delta\mathbf{u}(k) = \left[\mathbf{T}(q^{-1})\mathbf{z_r}(k+d) - \mathbf{S}(q^{-1})\mathbf{y}(k)\right] \tag{9}$$

where

$$\mathbf{S}(q^{-1}) = \mathbf{S}_o + \mathbf{S}_1 q^{-1} + \cdots + \mathbf{S}_{na} q^{-na} \tag{10}$$

is the polynomial equation solution of the following equation:

$$\mathbf{T}(q^{-1})\boldsymbol{\xi}(q) = \mathbf{E}(q,q^{-1})\mathbf{A}(q^{-1})\Delta + \mathbf{S}(q^{-1}) \tag{11}$$

and

$$\mathbf{R}(q^{-1}) = \mathbf{R}_o + \mathbf{R}_1 q^{-1} + \cdots + \mathbf{R}_{nb+d} q^{-nb+d} \tag{12}$$

is given by

$$\mathbf{R}(q^{-1}) = q^{-1}\mathbf{E}(q,q^{-1})\mathbf{B}(q^{-1}) + \mathbf{P}(q^{-1}) \tag{13}$$

where $\mathbf{E}(q,q^{-1})$ is a polynomial matrix given by

$$\mathbf{E}(q,q^{-1}) = \sum_{i=1}^{d} \mathbf{E}_i q^i \tag{14}$$

The control law, Eq. (9), ensures offset free for the control system (zero tracking error) when the following conditions are guaranteed:

Lemma 1. *Consider the polynomial matrix* $\mathbf{P}(q^{-1})$ *defined by*

$$\mathbf{P}(q^{-1}) = \mathrm{diag}(p^{(1)}(q^{-1}), \ldots, p^{(n)}(q^{-1})) \tag{15}$$

where

$$p^{(i)}(q^{-1}) = p_o^{(i)} + p_1^{(i)} q^{-1} + \cdots + p_{np}^{(i)} q^{-np} \tag{16}$$

and the control matrix $\mathbf{B}(q^{-1})$ *is such that* $\mathbf{P}(q^{-1})\mathbf{B}(q^{-1}) = \mathbf{B}(q^{-1})\mathbf{P}(q^{-1})$. *The closed-loop system matrix is expressed by*

$$\begin{aligned}\left[q^{-1}\mathbf{B}(q^{-1})\mathbf{T}(q^{-1})\boldsymbol{\xi}(q) + \mathbf{P}(q^{-1})\mathbf{A}(q^{-1})\Delta\right]\mathbf{y}(k) \\ = q^{-1}\mathbf{B}(q^{-1})\mathbf{T}(q^{-1})\boldsymbol{\xi}(q)\mathbf{y}_r(k)\end{aligned} \tag{17}$$

Consider

$$\mathbf{C}(q^{-1}) = q^{-1}\mathbf{B}(q^{-1})\mathbf{T}(q^{-1})\xi(q) + \mathbf{P}(q^{-1})\mathbf{A}(q^{-1})\Delta \tag{18}$$

a Schur polynomial and $(\mathbf{P}(q^{-1}), \mathbf{T}(q^{-1}))$, $(\mathbf{A}(q^{-1}), \mathbf{T}(q^{-1}))$, $(\mathbf{B}(q^{-1}), \mathbf{P}(q^{-1}))$ *have no common roots outside of the unit disk.*

If the determinant of $\mathbf{C}(q^{-1})$ *is given by*

$$\mathbf{W}(q^{-1}) = w_o + w_1 q^{-1} + \cdots + w_n q^{-n} \tag{19}$$

then, the stability can be evaluated by the Jury criterion (Scattolini & Bittanti, 1990) *and, for a stable system, the following condition must be ensured*:

$$|w_o| > \sum_{i=1}^{n} |w_i| \tag{20}$$

Also, if the condition $\|D\|_\infty < 1$ *is ensured, then all roots of* $\mathbf{W}(q^{-1})$ *lie strictly inside the unit circle in the z-plane* (Ngo & Erickson, 1997), *where* $D = [w_o\ w_1\ \ldots\ w_n]$. □

The performance of the control law, Eq. (9), can be improved by adding an auxiliary input and, therefore, connecting the GMV controller with a VSC based block (Corradini & Orlando, 1995; Furuta, 1993; Wang et al., 2011, 2013). The proposed multivariable VSC/GMV control algorithm is based on the following theorem:

Theorem 1. *Given a system S of the form (1), the following incremental control law*

$$\begin{aligned}\mathbf{R}(q^{-1})\Delta\mathbf{u}(k) = \mathbf{T}(q^{-1})\mathbf{z}_r(k+d) - \mathbf{S}(q^{-1})\mathbf{y}(k) \\ + \mathbf{s}(k) + \mathbf{v}(k)\end{aligned} \tag{21}$$