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# Zero-phase velocity tracking of vibratory systems

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ABSTRACT

The focus of this paper is on the development of an input shaper/time-delay filter that permits the precise tracking of a ramp input, while eliminating residual vibrations. Zero phase error velocity tracking is often required in applications where moving parts have to be mated, such as manufacturing lines with high production output. A closed form solution to a pre-filtering technique is presented which achieves the desired characteristics. The performance of this technique is compared with other input shaper designs in current literature, and is shown to achieve smaller settling time and maintain zero steady state phase error without a priori knowledge of the initiation and termination of ramp profiles. The technique is then physically applied to a rotary pendulum to demonstrate the consistency of its ramp tracking and vibration reduction capabilities.

## 1. Introduction

Input shaping is a feedforward control technique that has been used extensively to eliminate residual vibrations for systems undergoing rest-to-rest maneuvers (Singer & Seering, 1990; Singh & Vadali, 1993a; Sorensen, Singhose, & Dickerson, 2007). This is achieved by convolving the reference step input with a sequence of impulses that cancel the oscillatory dynamics present in the target system. Additional impulses can be introduced into the sequence to improve robustness to uncertainties in the model parameters, at the cost of longer maneuver times. Various groups (Rhim & Book, 2004; Tzes & Yorkovich, 1993) have also developed adaptive input shaping schemes to improve the robustness while minimizing the duration of impulse sequence.

While traditional input shaping schemes have been well demonstrated for rest-to-rest maneuvers, there are instances where constant velocity tracking is required. Constant velocity motion is represented by a ramp signal, and the direct application of input shaping schemes introduces a non-zero steady state phase lag after the residual vibrations are eliminated. In some cases, this phase error is permitted as it does not interfere with operation requirements, such as in the control of high-speed electron microscopy scanner head (Croft & Devasia, 1999), wafer scanner (Butler, 2013), and high speed tape drives (Mathur & Messner, 1998). Masterson, Singhose, and Seering (2000) recognize the delay generated in completing a prescribed scan when an input shaper is used to eliminate residual vibrations. To satisfy the constraint imposed by the scan time, they study the effect of changing the scan velocity to compensate for the delay. For applications where zero velocity tracking error is crucial to the operation, traditional input shapers are no longer sufficient. Such requirement arises when compliant industrial robots are used in high throughput production lines. Kamel, Lange, and Hirzinger (2008) discussed the need for zero phase error tracking during mating operations between car wheels and the corresponding chassis moving with constant velocity on an assembly line. The wheels are handled by robotic end-effectors with built-in compliance to avoid damaging the chassis. The same compliance also introduces oscillations that are detrimental to the alignment, and input shapers are used to minimize the vibrations.

Attempts to reduce or eliminate ramp tracking error have been suggested in Masterson et al. (2000), Tomizuka (1987), Butterworth, Pao, and Abramovitch (2008), and Kamel et al. (2008). Masterson et al. (2000) developed a procedure for constant velocity scanning with flexible sensors by increasing the reference scan velocity to compensate for the phase lag due to input shapers, while maintaining the total scan time. Tomizuka (1987) proposed a zero phase error tracking algorithm, which relies on a priori knowledge of the trajectory and is often referred to as a model inversion based technique. Butterworth et al. (2008) compared the performance of model inversion techniques on velocity tracking of an atomic force microscope which is characterized by non-minimum phase behavior. Dynamic inversion assuming an output with a desired smoothness permits identifying a bounded smooth input which has been shown to track a reference profile by Piazzi and Visioli for the end point control of a flexible link (Piazzi & Visioli, 2011) and for the control of an overhead crane (Piazzi & Visioli, 2002). Dynamic inversion has also been demonstrated to work well for nonlinear systems with affine input where the number of inputs and outputs is the same

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(Devasia, Chen, & Paden, 1996). Kamel et al. (2008) described several methods for input shaping with predictive path scheduling for low sampled systems in their efforts to damp oscillations in the robot end effector. In all these cases, their designs are acausal and require knowledge of the maneuver trajectory.

In this paper, a simple casual technique consisting of a shaped ramp profile in conjunction with a shaped step profile is shown to achieve precise ramp signal tracking. Traditional input shaping schemes are readily applicable within the ramp-following framework, allowing for improvements in robustness, as well as designs for multi-mode systems in both the continuous and discrete domain. Closed-form solutions are also available, allowing for efficient implementations. Finally, the technique is applied to the velocity tracking of a rotary pendulum for experimental validation.

## 2. Time-delay filter/input shaper

Input shapers (IS) and time-delay filters (TDF) are pre-filtering techniques that are often used to eliminate residual vibrations in rest-to-rest maneuvers. The IS design is derived in the time-domain while the TDF is designed in the frequency domain. The terms IS and TDF are used interchangeably in this paper, and in figures they are labeled based on the employed design methods. Time-delay filter relies on canceling the under-damped poles of the system with zeros of the time-delay filter transfer function. This section will briefly review the results from traditional TDF design that are immediately applicable to the development of the ramp-following time delay filters (RF-TDF), the reader is referred to Singh (2010) for a more comprehensive treatment.

The general structure of a time-delay filter is shown in Fig. 1. Specifically for a second order under-damped system

$$G(s)=\frac{Y(s)}{U(s)}=\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}, \tag{1}$$

a minimum of two terms in the time-delay filter $P(s)$ is required to cancel the poles of the system. The single-delay TDF ($N=2$) therefore assumes the form

$$P(s)=\sum_{i=0}^{N-1}A_i e^{-sT_i}=A_0+A_1e^{-sT_1}, \tag{2}$$

which is set to zero at the system poles $s=-\zeta\omega+j\omega\sqrt{1-\zeta^2}$, to obtain a closed form solution for the parameters of the time-delay filter:

$$A_0=\frac{e^{\zeta\pi/\sqrt{1-\zeta^2}}}{1+e^{\zeta\pi/\sqrt{1-\zeta^2}}},\quad A_1=\frac{1}{1+e^{\zeta\pi/\sqrt{1-\zeta^2}}},\quad T=\frac{\pi}{\omega\sqrt{1-\zeta^2}}. \tag{3}$$

The resulting filter corresponds exactly to the solution of the posicast controller (Smith, 1957) and the zero-vibration (ZV) input shaper (Singer & Seering, 1990). The residual vibrations after the final maneuver time $T_{N-1}$ is completely eliminated when the system parameters are known exactly. It is important to note that the DC gain of any time-delay filter must be unity, i.e.

$$\sum_{i=0}^{N-1}A_i=1 \tag{4}$$

to ensure that the output amplitude is the same as the input amplitude of the step after the final maneuver time. Fig. 2 shows the response of a second order system to the filtered step input.

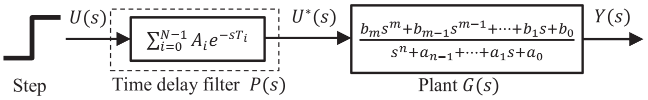

**Fig. 1.** Traditional time-delay filter structure.

### 2.1. Robust design (TDF)

The performance of the time-delay filter depends on the knowledge of the system model. Since the parameters of the system model are seldom known exactly in practice, there is a need to synthesize TDF that are insensitive to errors in these parameters.

#### 2.1.1. Cascade design

Singh and Vadali (1993b) have shown that by placing multiple zeros of the time-delay filter at the nominal location of the uncertain poles of the system, one can achieve robustness in the proximity of the nominal model. For instance, by cascading two single TDF, the robust TDF with two delays is given as

$$P(s)=(A_0+A_1e^{-sT_1})^2=A_0'+A_1'e^{-sT_1}+A_2'e^{-s2T_1} \tag{5}$$

where $A_0$, $A_1$ and $T_1$ are the same as (2). Evidently from the above equation, the final maneuver time is now $2T_1$, i.e. the increase in robustness also increases the maneuver time. The formulation is equivalent to adding an additional constraint forcing the derivatives of the TDF at the nominal frequency or damping ratio to be also zero (Singh & Vadali, 1993b). This is sometimes referred to as zero vibration and derivative (ZVD) input shaper (Singer & Seering, 1990). Higher derivatives can also be forced to zero by cascading additional single-delay TDF, which further improves the robustness while increasing maneuver time. Larger maneuver time is often undesirable in high-speed precision applications, thus the trade-off between robustness and maneuver time must be carefully considered. For the remainder of this paper, the parameterization given by (5) will be simply referred to as the robust TDF.

#### 2.1.2. Minimax design

TDF with 2 delays introduces robustness based on the knowledge of nominal model parameters. Alternatively, a minimax TDF can be designed to improve the robustness within a domain of uncertainty (Singh, 2010). Since the minimax TDF is a numerical optimization based technique, only a special case is considered here such that its applicability to the ramp-following time delay filter can be assessed in subsequent sections.

The minimax TDF is designed for the same second order system in (1) subjected to a unit step input. A uniform distribution for the uncertainty in the natural frequency is assumed. The optimization problem can be stated as

$$\min_{A_i,T_i}\max_{\omega}\frac{1}{2}\dot{y}_f^2+\frac{1}{2}\omega^2(y_f-y_{ref,f})^2 \tag{6}$$

subject to the following constraints on the TDF parameters:

$$\sum_{i=0}^{N-1}A_i=1 \tag{7}$$

$$0<T_{i-1}<T_i, \tag{8}$$

where the subscript $f$ denotes value of the variable immediately after the final maneuver time $T_{N-1}$. The cost function (6) measures the residual energy of the system response for normalized mass $m=1$, with corresponding stiffness $\omega^2$. The uncertain domain over $\omega$ is discretized to produce a set of plant models. A unit step input is shaped by the candidate time-delay filter, and the residual energy is evaluated for each plant model. The maximum residual energy over the specified range of uncertain $\omega$ is minimized, resulting in a desensitized time-delay filter design. A two-delay TDF ($N=3$) is used here, and the minimax problem is solved with MATLAB's `fminimax` function to obtain the parameters $A_0$, $A_1$, $A_2$, $T_1$ and $T_2$. While minimax design based on the present cost function has been shown to reduce residual vibrations effectively for rest-to-rest maneuvers (Singh, 2002), it will be illustrated in