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Construction of adhesion maps for contacts between a sphere and a half-space: Considering size effects of the sphere

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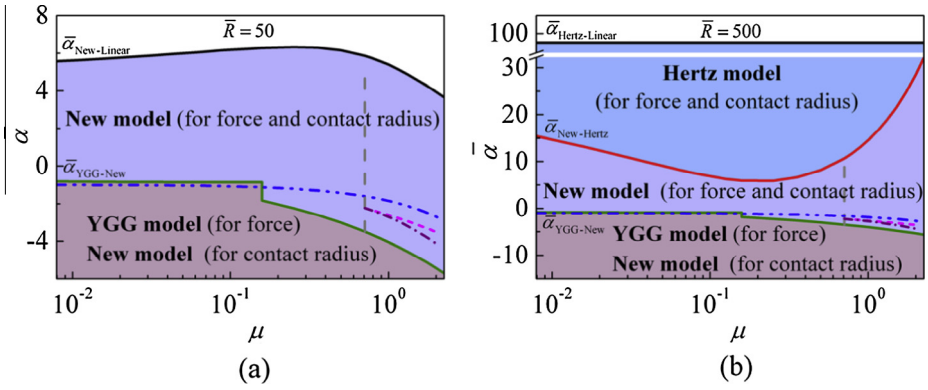
GRAPHICAL ABSTRACT



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ABSTRACT

Previous adhesion maps, such as the JG (Johnson–Greenwood) and YCG (Yao–Ciavarella–Gao) maps, are used to guide the selection of Bradley, DMT, M-D, JKR and Hertz models. However, when the size of the contact sphere decreases to the small scale, the applicability of JG and YCG maps is limited because the assumptions regarding the contact region profile, interaction between contact bodies and sphere shape in the classical models constituting these two maps are no longer valid. To avoid this limitation, in this paper, a new numerical model considering size effects of the sphere is established first and then introduced into the new adhesion maps together with the YGG (Yao–Guduru–Gao) model and Hertz model. Regimes of these models in the new map under a certain sphere radius are demarcated by the criteria related to the relative force differences and the ratio of contact radius to sphere radius. In addition, the approaches at pull-off, jump-in and jump-out for different Tabor parameters and sphere radii are provided in the new maps. Finally, to make the new maps more feasible, the numerical results of approaches, force and contact radius involved in the maps are formularized by using the piecewise fitting.

1. Introduction

With the continuous miniaturization of devices, such as micro/nanoelectro-mechanical systems (MEMS/NEMS), adhesive contact problems induced by molecular interactions become remarkable [1]. To guide the selection of different models for the

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adhesive contact between spheres or between a sphere and a half-space, adhesion maps have been constructed. The first map, often referred to as the JG (Johnson–Greenwood) adhesion map, was constructed by Johnson et al. [2] who used the dimensionless force $F/(\pi R\Delta\gamma)$ and elasticity parameter $\lambda = 1.16\mu$ to demarcate the application ranges of the classical Hertz [3], Bradley [4], DMT [5], M-D [6] and JKR [7] models. μ is the well-known Tabor parameter [8] which is expressed as $[R\Delta\gamma^2/(E^{*2}\varepsilon^3)]^{1/3}$ (where R is the effective radius of spheres, $\Delta\gamma$ is the work of adhesion, E^* is the effective elastic modulus and ε is the equilibrium distance between two surfaces) and represents the ratio of elastic deformation energy to surface energy. Yao et al. [9] found that the JG adhesion map only included the compressive force range ($F \geqslant 0$). To replenish the original JG map in the tensile force range ($F < 0$), they extended the borders between Bradley, DMT, M-D and JKR regimes and then modified the application ranges of these models by a parameter "strength limit". Through this method, another adhesion map named as the YCG (Yao–Ciavarella–Gao) map was constructed. However, the use of the strength limit may not be appropriate as the combination of a given strength limit and an unexceptional Tabor parameter may result in an unrealistic sphere radius [10].

In essence, both the JG map and the YCG map are approximate ones as the classical models constituting them are derived analytically resting on two kinds of basic assumptions. The first kind is that the contact region in Hertz, DMT, M-D and JKR models is perfectly flat. Strictly speaking, this assumption can never be valid as the surface gap has a nonlinear relationship with the interaction force between contact bodies [10], but it could be appropriate in cases of larger Tabor parameters and higher loads which cause significant deformations. The second kind is that the sphere radius is large enough so that the shape of the sphere can be represented by a paraboloid and the adhesion between two contact solids can be represented by the interaction between their adjacent surfaces. To be specific, the interaction between surfaces in the Hertz, Bradley, DMT, M-D and JKR models is described by a function with only repulsive surface forces, a Lennard–Jones surface force function ignoring the deformation, a step function with long-range surface forces, a Dugdale function and a delta-like function with short-range surface forces, respectively. Due to this kind of assumptions, it can be deduced that under the premise of larger sphere radius, the Bradley model is valid only when the external force is sufficiently light, while the Hertz, DMT, M-D and JKR models are valid when the external force is sufficiently high. When the sphere radius decreases to the small scale, the paraboloidal approximation is not valid and the adhesion between contact solids can no longer be described by the interaction between surfaces because the solid itself may become completely immersed in the interaction zone [11].

Evidently, the above two kinds of assumptions in the classical contact models have limited the applicability of the JG and YCG maps. New adhesion maps which can be applied to contact problems with size ranging from micro- to nano-scales need to be constructed. To do this, three problems should be solved. First, a new adhesion model which can not only eliminate the limitations but also assess the applicability of classical contact models is desired. Second, results of the classical contact models should be compared with those of the new model so that the models which are not only appropriate but also as simple as possible for constructing the new adhesion maps can be selected and the criteria for demarcating the application ranges of these models can be determined. Third, the specific boundaries between the selected models need to be calculated at different Tabor parameters and sphere radii and restricted by the linear elasticity theory. After solving these problems, new adhesion maps can be constructed. In order to make the new adhesion maps more convenient for application, curve fitting equations for the critical approaches describing the boundaries between different models and the force and contact radius of the new model are preferred compared with the numerical results.

2. Adhesion model based on Hamaker summation method

As discussed in the introduction, a new adhesion model which can eliminate the limitations in classical contact models is needed. To establish this model, two previous findings are considered. First, numerical solving should be conducted if nonlinear relationships between deformations and forces are taken into account [12]. Second, the Hamaker summation method [13] which obtains the interaction potential between two solids by summing up the intermolecular potential through the whole volumes is an appropriate theory to the adhesive contact problems at both micro- and nano-scales [14]. Therefore, a new numerical adhesion model based on Hamaker summation method is established in this section. An outline of this model is presented in the following.

Fig. 1 is a schematic drawing of the adhesive contact between an elastic half-space 1 and a rigid sphere 2 with radius R, in which d is the distance between the bottom of the sphere and the half-space expressed as

$$d(x,y) = -\alpha + u(x,y) \tag{1}$$

where α is the approach between the rigid sphere and the undeformed half-space. The elastic compression δ in the classical contact models [3–7] can be related to the approach α by $\delta = \alpha + \varepsilon$.

$u(x,y)$ denotes the elastic deformation of the half-space and can be calculated by using the Boussinesq integral [15]

$$u(x,y) = \frac{1}{\pi E^*}\iint \frac{p(\xi,\eta)\mathrm{d}\xi\mathrm{d}\eta}{\sqrt{(x-\xi)^2+(y-\eta)^2}} \tag{2}$$

where $1/E^* = (1-\nu_1^2)/E_1$. E_1 and ν_1 denote the Young's modulus and Poisson's ratio of half-space 1, respectively. $p(x,y)$ is the local pressure distribution caused by molecular interactions between the sphere and the half-space. $p > 0$ when the pressure is compressive, while $p < 0$ when the pressure is tensile.

The total interaction force between the sphere and the half-space can be obtained by integrating the intermolecular potential $w(s)$ with all molecules in the body and converting the double volume integral into the effective surface integral [14], which can be written as

$$\begin{aligned}\mathbf{A} &= \rho_1\rho_2\int_{V_2}\int_{V_1}\nabla_2 w(s)\mathrm{d}V_1\mathrm{d}V_2 \\ &= \rho_1\rho_2\int_{S_2}\int_{S_1}\mathbf{n}_2(\mathbf{G}\cdot\mathbf{n}_1)\mathrm{d}S_1\mathrm{d}S_2 = \int_{S_1}(\mathbf{b}\cdot\mathbf{n}_1)\mathrm{d}S_1\end{aligned} \tag{3}$$

where ρ_1, ρ_2 and $\mathbf{n}_1$, $\mathbf{n}_2$ are the number densities and surface unit normal vectors, respectively. V_1, V_2 and S_1, S_2 denote the volume

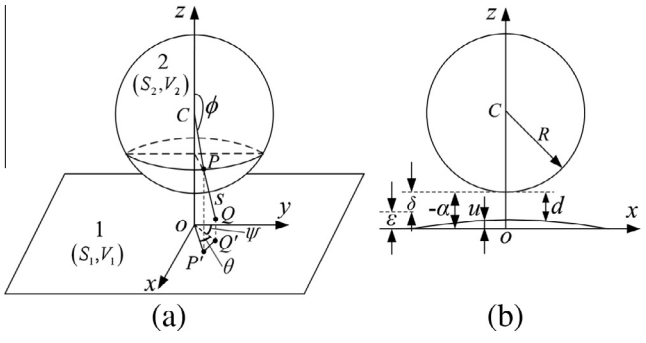

Fig. 1. Schematic of the adhesive contact between a rigid sphere and an elastic half-space. (a) 3D plot and (b) Cross-sectional profile.