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# Adaptive-gain second-order sliding mode observer design for switching power converters

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ABSTRACT

In this paper, an adaptive-gain, Second Order Sliding Mode (SOSM) observer for multi-cell converters is designed by considering it as a type of hybrid system. The objective is to reduce the number of voltage sensors by estimating the capacitor voltages from measurement of the load current. The proposed observer is proven to be robust in the presence of perturbations with *unknown* boundaries. As the states of the system are only partially observable, a recent concept known as $Z(T_N)$-observability is used to address the switching behavior. Multi-rate simulation results demonstrate the effectiveness and the robustness of the proposed observer with respect to output measurement noise and system uncertainty (load variations).



## 1. Introduction

In recent years, industrial applications requiring high power levels have used medium-voltage semiconductors (Gerry, Wheeler, & Clare, 2003; Meynard & Foch, 1992; Rech & Pinheiro, 2007; Rodriguez, Lai, & Peng, 2002). Because of the efficiency requirements, the power of the converter is generally increased by boosting the voltage. However, medium-voltage switching devices are not available. Even if they did exist, the volume and the cost of such devices would be substantial (Gateau, Fadel, Maussion, Bensaid, & Meynard, 2002). In this sense, the topology of multi-level converters, which have been studied during the last decade, becomes attractive for high voltage applications (Meynard & Foch, 1992). From a practical point of view, the series of a multi-cell chopper designed by the LEEI (Toulouse, France) (Bensaid & Fadel, D., 2001) leads to a safe series association of components working in a switching mode. This structure offers the possibility of reducing the voltage constraints evenly among each cell in a series. These lower-voltage switches result in lower conduction losses and higher switching frequencies. Moreover, it is possible to improve the output waveforms using this structure (Bensaid & Fadel, M., 2001; Bensaid & Fadel, 2002; Gateau et al., 2002). These flying capacitors have to be balanced to guarantee the desired voltage values at the output, which ensures that the maximum benefit from the multi-cell structure is obtained (Meynard, Fadel, & Aouda, 1997). These properties are lost if the capacitor voltage drifts far from the desired value (Bejarano, Ghanes, & Barbot, 2010). Therefore, a suitable control of the switches is required to generate the desired values of the capacitor voltages. The control of switches allows the current harmonics at the cutting frequency to be canceled and the ripple of the chopped voltage to be reduced (Defoort, Djemaï, Floquet, & Perruquetti, 2011; Djemaï, Busawon, Benmansour, & Marouf, 2011).

Several control methods have been proposed for multi-cell converters, such as nonlinear control based on input–output linearization (Gateau et al., 2002), predictive control (Defaÿ, Llor, & Fadel, 2008), hybrid control (Bâja, Patino, Cormerais, Riedinger, & Buisson, 2007), model predictive control (Defaÿ et al., 2008; Lezana, Aguilera, & Quevedo, 2009) and sliding mode control (Amet, Ghanes, & Barbot, 2011; Djemaï et al., 2011; Meradi, Benmansour, Herizi, Tadjine, & Boucherit, 2013). However, most of these techniques require measurements of the voltages of the capacitors to design the controller. That is, extra voltage sensors are necessary, which increases the cost and the complexity of the system. Hence, the estimation of the capacitor voltages using an observer has attracted great interest (Besançon, 2007).

It should be noted that the states of the multi-cell system are only partially observable because the observability matrix never has full rank (Besançon, 2007). Hence, the observability matrix rank condition cannot be employed in an observability analysis of a hybrid system such as the one considered here (Babaali & Pappas, 2005; Vidal, Chiuso, Soatto, & Sastry, 2003). A recent concept, $Z(T_N)$-observability (Kang, Barbot, & Xu, 2009), can be used to analyze the observability of a switched hybrid system and is applied in this work because the observability of the converter depends upon the switching control signals. Various observers have been designed for the multi-cell converters based on concepts such as homogeneous finite-time observers (Defoort et al., 2011), super-twisting sliding

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mode observers (Bejarano et al., 2010; Ghanes, Bejarano, & Barbot, 2009) and adaptive observers (Bejarano et al., 2010). The concept of observability presented in Kang et al. (2009) gives the condition under which there exists a hybrid time trajectory that makes the system observable. Using this concept, estimates of the capacitor voltages can be obtained from the measurements of the load current and the source voltage by taking advantage of the appropriate hybrid time trajectories. In the works of Ghanes et al. (2009) and Bejarano et al. (2010), based on a set of $p-1$ linearly independent equations with respect to the voltages in the $p-1$ capacitors, an algorithm is proposed to estimate the capacitor voltages which employs the pseudo-inverse of a matrix whose elements are the switching signals correspondingly.

In this paper, an observability analysis based on the results of Kang et al. (2009) and Bejarano et al. (2010) is performed for the multi-cell converter assuming measurements of the load current and the source voltage under certain conditions of the switching input sequences. Then, a novel adaptive-gain SOSM observer for multi-cell converters is introduced that takes into account certain perturbations (load variations) in which the boundaries of their first time derivatives are *unknown*. The proposed adaptive-gain SOSM algorithm combines the nonlinear term of the super-twisting algorithm (ST) and a linear term, the so-called SOSML algorithm (Moreno & Osorio, 2008). The behavior of the ST algorithm near the origin is significantly improved compared with the linear case. Conversely, the additional linear term improves the behavior of the ST algorithm when the states are far from the origin. Therefore, the SOSML algorithm inherits the best properties of both the linear and the nonlinear terms. An adaptive law of the gains of the SOSML algorithm is derived via the so-called "time scaling" approach (Respondek, Pogromsky, & Nijmeijer, 2004). The output observation error and its first time derivative converge to zero in finite time with the proposed SOSML observer such that the equivalent output-error injection can be obtained directly. Finally, the resulting reduced-order system is proven to be exponentially stable. That is, the estimates of the capacitor voltages, which are considered as the states of the observer system, converge to the real states exponentially. The main advantages of this paper are as follows:

- The estimates of the capacitor voltages are obtained directly through analyzing the information of the equivalent output-error injection.
- Only one parameter of the proposed SOSML algorithm has to be tuned.
- There are no a priori requirements on the perturbation bounds and the finite time convergence of the output error dynamics is proven via Lyapunov analysis.

This paper is organized as follows. In Section 2, a model of the multi-cell converter and its characteristics are presented. In Section 3, the observability of the multi-cell converter is studied with the concept of $Z(T_N)$-observability. Section 4 discusses the design of the proposed adaptive-gain SOSML observer for estimating the capacitor voltages. Section 5 gives multi-rate simulation results including a comparison with a Luenberger switched observer with disturbances.

## 2. Modeling of the multi-cell converter

The structure of a multi-cell converter is based on the combination of a certain number of cells. Each cell consists of an energy storage element and commutators (Gateau et al., 2002). The main advantage of this structure is that the spectral quality of the output signal is improved by a high switching frequency between the intermediate voltage levels (McGrath & Holmes, 2007). An instantaneous model that was presented in Gateau et al. (2002) and describes fully the hybrid behavior of the multi-cell converter is used here.

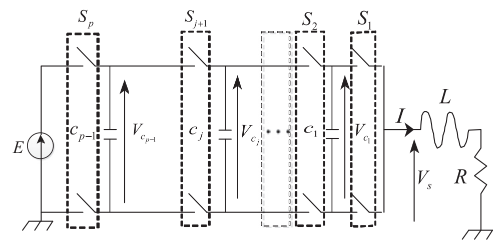


**Fig. 1.** Multi-cell converter on RL load.

Fig. 1 depicts the topology of a converter with $p$ independent commutation cells that is connected to an inductive load. The current $I$ flows from the source $E$ to the output through the various converter switches. The converter thus has a hybrid behavior because of the presence of both discrete variables (the switching logic) and continuous variables (the currents and the voltages).

Through circuit analysis, the dynamics of the $p$-cell converter were obtained as in the following differential equations:

$$\begin{cases} \dot{I} = -\dfrac{R}{L}I + \dfrac{E}{L}S_p - \displaystyle\sum_{j=1}^{p-1}\dfrac{V_{c_j}}{L}(S_{j+1}-S_j), \\ \dot{V}_{c_1} = \dfrac{I}{c_1}(S_2 - S_1), \\ \vdots \\ \dot{V}_{c_{p-1}} = \dfrac{I}{c_p}(S_p - S_{p-1}), \end{cases} \tag{1}$$

where $I$ is the load current, $c_j$ is the $j$th capacitor, $V_{c_j}$ is the voltage of the $j$th capacitor and $E$ is the voltage of the source. Each commutation cell is controlled by the binary input signal $S_j \in \{0,1\}$, where $S_j=1$ indicates that the upper switch of the $j$th cell is on and the lower switch is off and $S_j=0$ indicates that the upper switch is off and the lower switch is on. The discrete inputs are defined as follows:

$$\begin{cases} u_j = S_{j+1} - S_j, \quad j = 1, \dots, p-1 \\ u_p = S_p. \end{cases} \tag{2}$$

With Eq. (2), the system (1) can be represented as follows:

$$\begin{cases} \dot{I} = -\dfrac{R}{L}I + \dfrac{E}{L}u_p - \displaystyle\sum_{j=1}^{p-1}\dfrac{V_{c_j}}{L}u_j, \\ \dot{V}_{c_1} = \dfrac{I}{c_1}u_1, \\ \vdots \\ \dot{V}_{c_{p-1}} = \dfrac{I}{c_{p-1}}u_{p-1}, \\ y = I. \end{cases} \tag{3}$$

Assuming that only the load current $I$ can be measured, it is easy to represent the system (3) as a hybrid (switched affine) system:

$$\begin{cases} \dot{x} = f(x,u) = A(u)x + B(u), \\ y = h(x,u) = Cx, \end{cases} \tag{4}$$

where $x = [I\ V_{c_1} \cdots V_{c_{p-1}}]^T$ is the continuous state vector, $u = [u_1\ u_2 \cdots u_p]^T$ is the switching control signal vector which takes