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Distributed identification and control of spatially interconnected systems with application to an actuated beam

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ABSTRACT

This paper presents a case study on modelling and control of spatially interconnected systems. Considered is a vibration control problem, with experimental results on a flexible beam that is equipped with an array of piezo sensors and actuators. The sensor–actuator array induces a spatial discretization of the beam into an array of interconnected subsystems. Models are experimentally identified that have the structure of spatially interconnected systems. Based on the identified models, distributed control schemes are designed by solving a linear matrix inequality (LMI) problem that has the size of a single subsystem. Modelling and control is considered for both spatially invariant and spatially varying systems; in the latter case the system is represented as linear parameter-varying (LPV) system that is scheduled not over time but over space. Simulation and experimental closed-loop results demonstrate the practicality and efficiency of the underlying framework.

1. Introduction

Modelling and control of distributed parameter systems is an area of significant practical importance that has been receiving considerable attention for many years. The dynamic properties of the class of distributed systems considered here are described by partial differential equations (PDEs); typical examples include temperature or concentration profiles that are to be controlled in chemical processes (Christofides, 2001), or deflection profiles of extended mechanical structures arising in vibration control problems (Tzou & Bergman, 1998).

Due to the spatial continuum, this class of systems is often referred to as infinite-dimensional systems, indicating the infinite dimension of the state space. The well-developed semigroup theory (Curtain & Zwart, 1995) has been widely employed for a precise mathematical treatment of the internal dynamics of an infinite-dimensional system, which is significantly more difficult than the finite-dimensional theory (see Bamieh, Paganini, & Dahleh, 2002; Jovanovic, 2004). As for the control in practice, the fast development of microelectromechanical systems (MEMS) and light-weight materials enables effective control without changing the nominal dynamics significantly. Spatially distributed piezoelectric patches have been often used for the control of flexible structures and fluid–structure systems (see, e.g., Meurer, 2013; Robu, Baudouin, & Prieur, 2009; Fleming & Reza Moheimani, 2004). Modelling and control of smart structures with built-in sensors and actuators has been studied in Tzou (1993) and Banks, Smith, and Wang (1996).

When – for the purpose of control – such a distributed system has been equipped with a spatially distributed array of sensor–actuator pairs, the sensor–actuator array induces a spatial discretization, resulting in a network of interconnected subsystems each with a sensor and actuator channel.

For controlling such networks an efficient theoretical framework was proposed in D'Andrea and Dullerud (2003), where the dynamic properties of the distributed system are captured by a multidimensional state space model that has the size of a single subsystem. An idealizing assumption required in this framework is the linear time- and space-invariant (LTSI) properties of the distributed system (which implies either infinite spatial extension or spatial periodicity). A modification of the approach that can be applied to linear time- and/or space-varying (LTSV) systems, using the idea of linear parameter-varying (LPV) systems with scheduling not only over time but also over space, was proposed in Wu (2003) and further refined in Liu, Hoffmann, and Werner (2013). A considerable practical advantage of the spatially interconnected systems approach is that it reduces the complexity of analysis and synthesis of distributed control schemes to the size of a single subsystem. Distributed controllers can be tuned to minimize the closed-loop induced $\mathcal{L}_2$ norm, while the synthesis problem can be solved as a linear matrix inequality (LMI) problem.

Designing distributed control schemes in this framework

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requires an accurate model of the distributed system to be controlled. Constructing such a model based on first principles is often difficult, and an attractive alternative is the experimental identification of a black-box model that possesses the desired localized structure of a spatially interconnected system. A least-squares approach – based on distributed input-output models – to the identification of spatially invariant interconnected systems was proposed in Ali, Chughtai, and Werner (2009), and extended in Ali, Chughtai, and Werner (2010a) and Ali, Ali, Abbas, and Werner (2011) to spatially varying systems. An alternative approach – based on subspace identification – was proposed for spatially invariant systems in Haber, Fraanje, and Verhaegen (2011).

In this paper, the potential of the spatially interconnected system approach for solving practical control problems is demonstrated, with its experimental application to a vibration control problem. An aluminium beam of 4.8 m length, equipped with 16 collocated pairs of piezo actuators and sensors, is used as a test bed. The experimental identification of suitable models and their validation are presented, together with experimental closed-loop results that demonstrate the performance of distributed control schemes. Identification and control experiments have been carried out under the assumptions of both spatial invariance and spatially varying properties, for the latter case some of the evenly spaced sensor–actuator pairs were deactivated to generate subsystems with heterogeneous dynamic properties.

Preliminary versions of the results presented here have been reported in a number of conference papers. In Liu and Werner (2013), data-driven distributed identification techniques developed in Ali et al. (2009, 2010b) have been experimentally implemented on the same test bed employed here for an LTSI model identification, and on a relatively short beam (measuring 0.75 m in length) for a distributed LPV model identification to capture the dominant boundary effects. Improved identification accuracy based on finite element (FE) modelling has been achieved in Liu, Gross, and Werner (2015). Provided the distributed framework and controller synthesis conditions developed in D'Andrea and Dullerud (2003) for LTSI models and in Wu (2003) for parameter-dependent interconnected systems in terms of constant Lyapunov functions (CLFs), their experimental validation has been demonstrated in Liu, Mendez Gonzalez, and Werner (2014). To reduce the conservatism induced by the use of CLFs, distributed LPV controller design for LTSV systems using parameter-dependent Lyapunov functions (PDLFs) has been proposed in Liu et al. (2013) with its performance illustrated using a simulation example. Furthermore, to account for the performance degradation due to actuator saturation, a two-step distributed anti-windup design has been presented in Liu and Werner (2015).

The aim of this paper is to unify previous results and provide a comprehensive overview of the experimental application of identification and control synthesis techniques in the framework developed in D'Andrea and Dullerud (2003) to a vibration control problem. Moreover, analysis and synthesis conditions for LTSV systems – first developed in Wu (2003) using CLFs, then extended in Liu et al. (2013) using PDLFs – are generalized here, which can accommodate the use of both CLFs and PDLFs for distributed LPV controller design. Finally, new simulation and experimental results that have not yet been reported are presented here, which include the closed-loop performance comparison

(i) using a centralized modal controller and a distributed controller;
(ii) using a distributed and a decentralized controller for LTSI systems;
(iii) using a distributed LPV controller for LTSV systems in terms of both CLFs and PDLFs.

Outline: Section 2 recaps the multidimensional state space models that define the plant and controller dynamics at the subsystem level. The test bed for experimental study is introduced as well. Section 3 revisits the mathematical models for distributed identification, and demonstrates the experimental validation on the test bed. State-space based system analysis and controller synthesis conditions are provided in Section 4, with the designed controllers evaluated in simulation and experimentally to suppress the vibratory motion of the actuated beam caused by the disturbance injection. Finally, conclusions are drawn in Section 5.

2. Preliminaries

This section reviews the multidimensional state space model that represents the distributed dynamics of a single subsystem interacting with its neighboring subsystems. It was first proposed in D'Andrea and Dullerud (2003) for LTSI systems, then extended to LTSV systems in Wu (2003). An experimental setup, that has been constructed to study spatially interconnected systems experimentally, is also introduced.

The framework employed here can be applied to problems with an arbitrary number of spatial dimensions. For the sake of presentation simplicity, and since the considered application is an actuated beam, in this paper only systems with a single spatial dimension are considered. All involved signals are functions of discrete time k and discrete space s, e.g., $x(k, s)$.

Definition 1 (*l_2 norm, D'Andrea and Dullerud, 2003*). Given a spatio-temporal vector-valued signal $x(k_0, s)$ with fixed time k_0, the l_2 norm is defined as

$$\|x(k_0, s)\|_{l_2}^2 := \sum_{s=-\infty}^{\infty} x^T(k_0, s)x(k_0, s). \tag{1}$$

The infinite spatial dimension indicates an infinite interconnection of subsystems, where the boundary condition effects need not to be concerned.

Definition 2 (*$\mathcal{L}_2$ norm, D'Andrea and Dullerud, 2003*). Given a spatio-temporal signal $x(k, s)$, its $\mathcal{L}_2$ norm is defined as

$$\|x(k, s)\|_{\mathcal{L}_2}^2 := \sum_{k=1}^{\infty}\sum_{s=-\infty}^{\infty} x^T(k, s)x(k, s). \tag{2}$$

Definition 3 (*System norm, D'Andrea and Dullerud, 2003*). The induced $\mathcal{L}_2$ norm of a two-dimensional operator $G: \mathcal{L}_2 \to \mathcal{L}_2$ is defined as

$$\|G\|_{\mathcal{L}_2} := \sup_{x\neq 0, x\in\mathcal{L}_2} \frac{\|Gx\|_{\mathcal{L}_2}}{\|x\|_{\mathcal{L}_2}}. \tag{3}$$

2.1. Spatially interconnected systems

The spatial discretization of a distributed system in one spatial dimension, such as an actuated beam, results in a string of interconnected subsystems. A spatially interconnected system in one spatial dimension with all involved signals listed is shown in Fig. 1. Depending on whether the subsystems are identical or exhibit varying dynamics, state space models that represent both LTSI and LTSV systems are introduced in this section.

2.1.1. LTSI plant and controller

Subsystems of LTSI spatially interconnected systems share identical dynamics. A multidimensional state space model (D'Andrea & Dullerud, 2003) represents the dynamics of a subsystem interacting with its neighboring subsystems as