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OWave control chart for monitoring the process mean

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ABSTRACT

In this paper a control chart for monitoring the process mean, called *OWave* (Orthogonal Wavelets), is proposed. The statistic that is plotted in the proposed control chart is based on weighted wavelets coefficients, which are provided through the Discrete Wavelets Transform using Daubechies *db*2 wavelets family. The statistical behavior of the wavelets coefficients when the mean shifts are occurring is presented, and the distribution of wavelets coefficients in the case of normality and independence assumptions is provided. The on-line algorithm of implementing the proposed method is also provided. The detection performance is based on simulation studies, and the comparison result shows that *OWave* control chart performs slightly better than Fixed Sample Size and Sampling Intervals control charts ($\overline{X}$, EWMA, CUSUM) in terms of Average Run Length. In addition, illustrative examples of the new control chart are presented, and an application to Tennessee Eastman Process is also proposed.

1. Introduction

Modern industrial systems are being more and more complex, and they are often affected by disturbances of the operating conditions, which then can lead to generate faults or assignable/special causes.

Statistical Process Control (SPC) aims to monitor processes in order to improve quality performance and/or ensure safety operation. SPC includes mainly the implementation of control charts, which are effective to detect assignable causes. Two parameters are generally monitored under the Gaussian model: the mean that represents the process target and the variance that characterizes the process dispersion. Several control charts, such as $\overline{X}$ control chart (Shewhart, 1925), EWMA (Roberts, 1959), and CUSUM (Page, 1954), have been provided to detect mean change. In order to improve the control charts detection performance, some parameters can be made variable, and thus providing adaptive control charts, such as Variable Sampling Intervals (VSI) control charts (Reynolds, Amin, Arnold, & Nachlas, 1988), Variable Sampling Size (VSS) control charts (Costa, 1994), and Variable Sample Size and Sampling Intervals (VSSI) control charts (Arnold & Reynolds, 2001; Costa, 1997). Tagaras (1998) presented an overview regarding the evolution of adaptive control charts. Recently, a comparison study was done to evaluate the effectiveness and robustness of nine control charts, for monitoring process mean (Ou, Wu, & Tsung, 2012). The authors concluded that, in terms of FSSI (Fixed Sample Size and Sampling Intervals), CUSUM/EWMA are the best charts, and the optimal SPRT (Sequential Probability Ratio Test) is the best chart in terms of adaptive control charts. Generally, adaptive control charts perform better than FSSI charts, but they are still more challenged for applications (Baxley, 1995).

In the last two decades, the association of wavelets analysis with statistical process control has been widely developed, especially in the multivariate SPC context. Multi-Scale SPC involves methods that combine the wavelet multi-scale decomposition and classical SPC techniques ($\overline{X}$, EWMA, CUSUM, PCA, etc.). Bakshi (1998) proposed a multi-scale monitoring methodology based on monitoring the reconstructed signals after thresholding the wavelets coefficients in the Principal Component Analysis (PCA) space. Bakshi (1999) and Aradhye, Bakshi, Strauss, and Davis (2003) have discussed the wavelet properties for monitoring process and concluded that the multi-scale methods are not as efficient as those dedicated to specific changes. Kano et al. (2002) compared several techniques such as dissimilarity measure (DISSIM) (Kano, Hasebe, Hashimoto, & Ohno, 2002), Multi-Scale Principal Component Analysis (MS-PCA) (Bakshi, 1998), and Moving-PCA (Kano et al., 2002). Through the benchmark Tennessee Eastman Process (TEP) (Downs & Vogel, 1993) they showed that MS-PCA technique is better than conventional methods in some cases of TEP faults and performs as conventional methods for other faults. In addition, Moving-PCA is also better than MS-PCA in some cases. The percentage of correct detection (detection reliability) was used as the performance indicator for these methods. Lu, Wang, and Gao (2003) highlighted the usefulness of wavelets analysis in order to detect faults with the same time behavior and different frequencies in the case of Three Tank Process and

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Tennessee Eastman Process. Yoon and MacGregor (2004) showed that multi-scale analysis might help us to isolate faults effectively, especially when the fault frequency is known (see also Reis & Saraiva, 2006). Lee, Park, and Vanrolleghem (2005) have proposed a methodology for monitoring batch processes; the multi-way PCA (Nomikos & MacGregor, 1994) was associated with wavelets analysis. The authors showed anew the benefit of wavelets analysis in order to improve the monitoring performance. Other associations using wavelets are still quite underdeveloped in the literature, such as multi-scale Partial Least Squares regression (PLS). A literature review regarding wavelet-based techniques for process monitoring is done by Ganesan, Das, and Venkataraman (2004).

Nevertheless, comparative studies are still unsatisfactorily presented in the literature, except those concerning the multi-scale PCA. Generally, the Haar wavelet is used in the field of MS-SPC and studies using other wavelets families are required as Aradhye et al. (2003) have noted. Moreover, there is a lack in the MS-SPC literature regarding the use of the Average Run Length (ARL) as the performance indicator of the proposed methodologies.

The main objective of this paper is to provide a new control chart based on weighted wavelets coefficients, in order to monitor the process mean. Daubechies *db*2 wavelet is used to generate wavelets coefficients, and the ARL is calculated in order to evaluate the performances of the proposed control chart.

This paper is organized as follows: the second section introduces the wavelets multi-scale decomposition and its distributional characteristics; we emphasize the ability of wavelets coefficients to detect mean change. The third section presents the theoretical aspects related to the construction of the proposed new control chart. Finally, in the last section conclusions and perspectives are presented.

2. Wavelets and detectability

Wavelets functions were introduced, at the first time by Jean Morlet in 1984, in the context of geophysical signal processing. Afterwards, wavelet theory has been developed profoundly (Daubechies, 1992; Mallat, 1989; Meyer, 1993). Today, wavelets are applied in different domains (Manufacturing Gao & Yan, 2010, Image and Signal Mallat, 1999; Meyer, 1993, Health Akay, 1998, Physics and Mechanics Fan & Zuo, 2006; Liang, Elangovan, & Devotta, 1998; Odgaard, Stoustrup, & Wickerhauser, 2006; Peng & Chu, 2004; Sun, Zi, & He, 2014; Yan, Gao, & Chen, 2014, etc.).

Wavelets functions are defined as follows:

$$\psi_{j,k}(t) = 2^{j/2}\Psi(2^j t - k) \tag{1}$$

where j represents the scale, k is the translation parameter, and Ψ is the mother wavelet.

The multiresolution concept (Mallat, 1989) proposes a framework to analyse signals with perfect reconstruction, in which wavelets coefficients, approximations $a_j(k)$ and details $d_j(k)$ are given across filter banks, as follows:

$$a_j(k) = \sum_{i=0}^{l} h[i]a_{j-1}[2k - i] \tag{2}$$

$$d_j(k) = \sum_{i=0}^{l} g[i]a_{j-1}[2k - i] \tag{3}$$

where $a_0 = x$ the original signal; $j, k \in Z$; l is the filter length; h and g are scaling and wavelets filters respectively.

In the following theorem, we present an original result regarding the parameters of the probability distribution of wavelets coefficients (details and approximations). This result will be used to design the *OWave* control chart.

Theorem 1. *Assume that* $X = [x_1, x_2, ..., x_n]$ *is a signal, where* x_i *are independent and identically distributed random variables as follows:* $x_i \rightsquigarrow \mathcal{N}(\mu_0, \sigma_0^2)$. *Consider Orthonormal and Biorthogonal compactly supported wavelets (Haar, Daubechies, Symlets, Coiflets, Discrete Meyer, Biorthogonal, Reverse Biorthogonal). The multiresolution analysis of X provides wavelets coefficients as follows:*

$$a_j(k) \rightsquigarrow \mathcal{N}\left(2^{j/2}\mu_0, \left(\sum_n h_n^2\right)^j \sigma_0^2\right)$$

$$d_j(k) \rightsquigarrow \mathcal{N}\left(0, \sum_n g_n^2\left(\sum_n h_n^2\right)^{j-1} \sigma_0^2\right)$$

Which are identically distributed random variables, and independent if the orthonormal wavelets are used, else they are slightly correlated.

The wavelets coefficients are summation of normally distributed variables consequently they follow the normal distribution (see Appendix). Furthermore, for orthonormal wavelets families the wavelets coefficients are independent at each scale. The independence of wavelets coefficients is a consequence of the projection into orthonormal bases. This is not the case for Biorthogonal bases where the correlation coefficient can easily be estimated by empirical study.

Here, the term *detectability* means the capacity of wavelets coefficients to detect mean changes in original data. From Theorem 1, we show that wavelets coefficients present some interesting distributional characteristics that reveal the original data features. Indeed, the approximation coefficients amplify the mean of the original data and may have a small variance, as in the case of Biorthogonal wavelets. Nevertheless, that may not be a useful approach since Biorthogonal coefficients are correlated. So in this case, in order to use effectively approximation wavelets to detect mean change one must take into account this autocorrelation. On the other side, details coefficients have a mean equal to zero. This characteristic could be useful to detect change in variance, because the mean change does not affect the details coefficients, as we have shown in our previous work (Cohen, Tiplica, & Kobi, 2016), in which we proposed *DeWave* control chart in order to monitor variance change.

In this paper, we propose a new control chart named *OWave* in order to monitor the process mean. Wavelets coefficients are provided through the Discrete Wavelet Transform using Daubechies *db*2 wavelet (Daubechies, 1992), which is an orthonormal wavelet. Its decomposition filters are defined as follows:

$$h_n = [-0.1294;\ 0.2241;\ 0.8365;\ 0.4830] \tag{4}$$

$$g_n = [-0.4830;\ 0.8365;\ -0.2241;\ -0.1294] \tag{5}$$

One can conclude from Theorem 1 that:

$$a_j(k) \rightsquigarrow \mathcal{N}(2^{j/2}\mu_0, \sigma_0^2) d_j(k) \rightsquigarrow \mathcal{N}(0, \sigma_0^2) \tag{6}$$

2.1. An illustrative example

In this example, we consider a window size of length L=8 observations/subgroups, as the smallest one that can be used rationally with the *db*2 wavelet, and in which the discrete wavelet transform is applied. Consequently, we get eight wavelets coefficients at the scale one (maximum decomposition level in this