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A nonlinear trajectory tracking controller for mobile robots with velocity limitation via fuzzy gains

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ABSTRACT

This paper proposes a fuzzy controller for trajectory tracking with unicycle-like mobile robots. Such controller uses two Takagi–Sugeno (TS) fuzzy blocks to generate its gains. The controller is able to limit the velocity and control signals of the robot, and to reduce the errors arising from its dynamics as well. The stability of the developed controller is proven, using the theory of Lyapunov. Experimental results show that the use of the proposed controller is attractive in comparison with the use of a controller with fixed saturation function.

1. Introduction

This work proposes a new approach to limit the control signals during a trajectory tracking with unicycle-like mobile robots. In the literature it is common to find works that use explicit saturation functions such as the hyperbolic tangent to limit control signals (Andaluz, Roberti, Toibero, & Carelli, 2012; Martins, Celeste, Carelli, Sarcinelli-Filho, & Bastos-Filho, 2008). In this work, however, fuzzy rules are adopted to achieve such limitation while keeping an efficient trajectory tracking controller operation.

The fuzzy control emerged in the 70s as a heuristic method based on the knowledge of the designer about the process to be controlled. Such method started being adopted after the publication of the works of Zadeh (1973) and Mamdani (1974). This methodology has the advantage of controlling a plant without an explicit knowledge of its dynamics. However, to prove the stability of the closed-loop control system using these controllers is a difficult task. In the field of mobile robots control, for instance, several works have used the heuristic methodology to design fuzzy controllers, without studying the system stability (Antonelli, Chiaverini, & Fusco, 2007; Deist & Fourie, 1993; Hung & Chung, 2006; Lakehal, Amirat, & Pontnau, 1995; Susnea, Filipescu, Vasiliu, & Filipescu, 2008). This means that there is no theoretical guarantee that the task being performed will be accomplished accordingly.

Due to the need of a formal proof of stability for the control system, fuzzy controllers have obtained a new focus with the Takagi–Sugeno (TS) fuzzy controllers (Takagi & Sugeno, 1985). Tanaka and Sugeno (1992) showed that the TS fuzzy controllers can be designed rigorously, following methodologies that can be reproduced consistently, guaranteeing the system stability and using several performance criteria. The technique most commonly adopted for the design of controllers represented as a TS fuzzy model is the Parallel Distributed Compensation (PDC) (Wang, Tanaka, & Griffin, 1996). This technique has been used successfully to design controllers for trajectory tracking with mobile robots (Guechi, Lauber, Dambrine, Klancar, & Blazic, 2010; Guechi, Abellard, & Franceschi, 2012) and to solve the backing control problem of a mobile robot with multiple trailers (Tanaka, Kosaki, & Wang, 1998). Nevertheless, it is important to emphasize that although the PDC technique is based on fuzzy models, this design methodology does not use the knowledge of the designer about the process.

The controller here proposed uses the control structure reported in Resende, Espinosa, Bravo, Sarcinelli-Filho, and Bastos-Filho (2011), combining the heuristic knowledge of the problem, the sector nonlinearity approach (Tanaka & Wang, 2001) and the inverse kinematic of the mobile platform. The use of the sector nonlinearity allows designing a fuzzy controller with a quite reduced number of rules and a quite low complexity, making it suitable for implementation in the low-profile processors generally available onboard mobile platforms.

Through the application of the inverse kinematic of the mobile platform, it was possible to design a TS fuzzy controller guaranteeing the stability of the closed loop system, but without using the PDC technique. More than this, it was possible to use the heuristic knowledge to reduce position errors caused by the difference

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between the desired values of linear and angular velocities (system inputs) and the current velocity values assumed by the mobile platform.

Three experiments run using a unicycle-like mobile robot are reported here, which have shown that the proposed controller performs better than the similar controller proposed by Martins et al. (2008). It is worth mentioning that the proposed controller can be adapted to other mobile platforms, demanding just the knowledge of its inverse kinematic.

To develop and validate the proposed controller, the paper is hereinafter organized in four sections. Section 2 presents the kinematic model of the unicycle-like mobile robot, while Section 3 details the proposed nonlinear controller with variable gains and discusses the system stability. In the sequel, Section 4 shows the experimental results and performance comparisons between the two aforementioned controllers. Finally, Section 5 highlights some conclusions.

2. The kinematic model adopted

Traditionally, in the motion control of unicycle-like mobile robots, the robot is considered as a point located at the middle of the virtual axle. In this work, however, the point that should follow a predetermined trajectory is located in front of the virtual axle (point d of Fig. 1). Such point is hereinafter named as the point of interest.

From Fig. 1, the velocity of the point of interest with respect to the inertial frame $\{I\}$ is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos\psi & -a\sin\psi \\ \sin\psi & a\cos\psi \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix} = \mathbf{C} \begin{bmatrix} u \\ \omega \end{bmatrix}, \tag{1}$$

where the linear velocity u and the angular velocity ω are the control inputs of the robot, $\dot{x}$ and $\dot{y}$ are, respectively, the velocity of the point of interest in the X and Y directions of the inertial frame, $a>0$ represents the distance between the point of interest and the center of the virtual axle, and ψ is the orientation of the robot, which is given by the solution of $\dot{\psi}=\omega$.

The appropriate values of u and ω to impose desired velocities $\dot{x}$ and $\dot{y}$ to the point of interest are determined by the inverse kinematics (Martins et al., 2008)

$$\begin{bmatrix} u \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\frac{1}{a}\sin\psi & \frac{1}{a}\cos\psi \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}, \tag{2}$$

where $\mathbf{C}^{-1}$ is the inverse kinematics matrix. Unlike the middle point of the virtual axle, the point of interest d does not have any velocity restriction in the robot workspace (such point can move in any direction).

3. The nonlinear trajectory tracking controller with fuzzy gains

3.1. The control law

During the trajectory tracking, the point of interest of the robot shall follow a programmed trajectory defined by an equation like $p(t)=(x_D(t), y_D(t))$, where (x_D, y_D) is the point to be followed and $t \geq 0$ is the time variable. To comply with this control objective, this work proposes the control law

$$\begin{bmatrix} u_r \\ \omega_r \end{bmatrix} = \mathbf{C}^{-1} \left(\begin{bmatrix} \dot{x}_D \\ \dot{y}_D \end{bmatrix} + \begin{bmatrix} \nu_x \\ \nu_y \end{bmatrix} \right), \tag{3}$$

where u_r and ω_r are the controller outputs which are, respectively, the linear and angular reference velocities; $\dot{x}_D$ and $\dot{y}_D$ are, respectively, the velocity of the programmed trajectory at the point (x_D, y_D) in the X and Y directions of the inertial frame; and ν_x and ν_y are the outputs of two "fuzzy velocity compensators" (FVC).

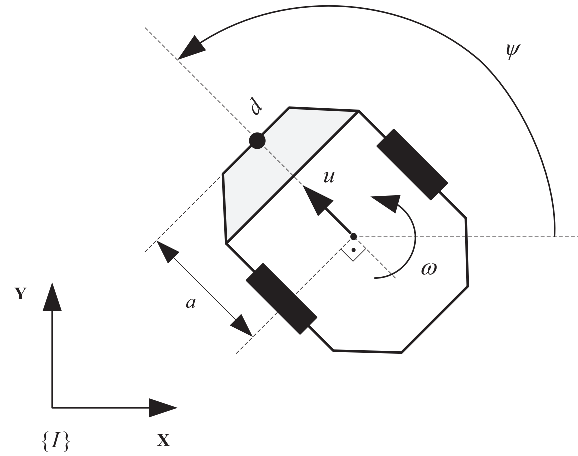

Fig. 1. The unicycle-like mobile robot and its kinematic parameters.

According to Fig. 2, the idea of the proposed controller is that once the point of interest d coincides with the desired point (x_D, y_D) at the trajectory, the reference velocities $\dot{x}_r$ and $\dot{y}_r$ are kept equal to the velocities of the reference trajectory, that is $\dot{x}_r = \dot{x}_D$, $\dot{y}_r = \dot{y}_D$, $\nu_x = 0$ and $\nu_y = 0$. Upon the occurrence of position errors $\tilde{x}$ and $\tilde{y}$, the fuzzy controller generates compensation terms for the velocities (ν_x and ν_y), until the point of interest d coincides with the desired point (x_D, y_D) at the trajectory again. Notice that the matrix $\mathbf{C}^{-1}$ is responsible for transforming $\dot{x}_r$ and $\dot{y}_r$ in u_r and ω_r.

The premise variables of the fuzzy velocity compensator X (FVC_X) are $|\dot{x}_D|$ and $|\tilde{x}|$, respectively the magnitude of the velocity of the programmed trajectory and the magnitude of the position error both in the X direction. In turn, the premise variables of the fuzzy velocity compensator Y (FVC_Y) are $|\dot{y}_D|$ and $|\tilde{y}|$, respectively the magnitude of the velocity of the programmed trajectory and the magnitude of the position error both in the Y direction.

The premise variables $|\tilde{x}|$ and $|\tilde{y}|$ are divided into three fuzzy sets: small error (S), medium error (M) and large error (B). The membership function of the small error fuzzy set is given by

$$f_S(|\tilde{e}|) = \begin{cases} 1, & |\tilde{e}| < \eta_1 \ [\text{m}]; \\ \dfrac{\eta_1}{(\eta_1-\eta_2)} - \dfrac{\eta_1 \cdot \eta_2}{(\eta_1-\eta_2)|\tilde{e}|}, & \eta_1 \leq |\tilde{e}| < \eta_2 \ [\text{m}]; \\ 0, & \eta_2 \leq |\tilde{e}| < \eta_3 \ [\text{m}]; \end{cases} \tag{4}$$

while the membership function of the medium error fuzzy set is given by

$$f_M(|\tilde{e}|) = \begin{cases} 0, & |\tilde{e}| < \eta_1 \ [\text{m}]; \\ \dfrac{\eta_1 \cdot \eta_2}{(\eta_1-\eta_2)|\tilde{e}|} - \dfrac{\eta_2}{(\eta_1-\eta_2)}, & \eta_1 \leq |\tilde{e}| < \eta_2 \ [\text{m}]; \\ \dfrac{\eta_2}{(\eta_2-\eta_3)} - \dfrac{\eta_2 \cdot \eta_3}{(\eta_2-\eta_3)|\tilde{e}|}, & \eta_2 \leq |\tilde{e}| < \eta_3 \ [\text{m}]; \end{cases} \tag{5}$$

and the membership function of the large error fuzzy set is given by

$$f_B(|\tilde{e}|) = \begin{cases} 0, & |\tilde{e}| < \eta_2 \ [\text{m}]; \\ \dfrac{\eta_2 \cdot \eta_3}{(\eta_2-\eta_3)|\tilde{e}|} - \dfrac{\eta_3}{(\eta_2-\eta_3)}, & \eta_2 \leq |\tilde{e}| < \eta_3 \ [\text{m}]; \end{cases} \tag{6}$$

where $|\tilde{e}|$ represents the magnitude of the position error ($|\tilde{x}|$ or $|\tilde{y}|$). Fig. 3 presents a sketch of such membership functions.

The premise variables $|\dot{x}_D|$ and $|\dot{y}_D|$ are divided into two fuzzy sets: low velocity (L) and high velocity (H). According to Fig. 4, the membership functions of such premise variables are defined