Control Engineering Practice 21 (2013) 1377–1385

Contents lists available at SciVerse ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Nonlinear joint state and parameter estimation: Application to a wastewater treatment plant

S. Bezzaoucha [a,b,*], B. Marx [a,b], D. Maquin [a,b], J. Ragot [a,b]

[a] Université de Lorraine, CRAN, UMR 7039, 2 avenue de la Forêt de Haye, Vandoeuvre-les-Nancy, Cedex 54516, France
[b] CNRS, CRAN, UMR 7039, France

ARTICLE INFO

Article history:
Received 13 November 2012
Accepted 7 June 2013
Available online 11 July 2013

Keywords:
Time-varying nonlinear system
Takagi–Sugeno model
Joint state and parameter observer
Waste water treatment plant

ABSTRACT

A systematic approach to joint state and time-varying parameter estimation for nonlinear systems is proposed in this paper. Applying the sector nonlinearity transformation to both the system nonlinearities and the time-varying parameters, the original system is equivalently rewritten as a Takagi–Sugeno system with unmeasurable premise variables. A joint state and parameter observer whose parameters are designed by solving an LMI optimization problem is then proposed. The target application is a realistic model of an activated sludge wastewater treatment plant, being an uncertain nonlinear system affected by a time-varying parameter.

1. Introduction

Since most of the control law and fault detection residual design (Basseville, 1998; Chen & Patton, 1999) are based on estimated state variables, the observer design for nonlinear systems can be viewed as the heart of system control and model-based diagnosis. Unfortunately, the introduction of time-varying parameters in the system models, needed to accurately represent the system behaviour, leads to more challenging problems in estimation. In this case, conventional observers, essentially developed for time invariant systems cannot be directly used, and so-called adaptive observers developed for joint state and unknown parameter estimation are needed (Zhang, 2002). The main difficulty in estimating the state of such systems comes from the lack of knowledge on the parameter evolution. In the present work, the authors focus on the nonlinear time-varying parameter systems where the parameters are inaccessible (non measurable) and may be considered as model disturbances, uncertainties or faults acting on the system evolution.

Some results have been published on the time-varying systems problem. For example, the state estimation of linear systems with unknown constant or time-varying parameter is respectively addressed in Zhang (2002) and Lubenova (1999). Extensions to nonlinear systems are proposed in Alcorta Garcia and Frank (1997), Besançon (2000), Rajamani and Hedrich (1995) and Zhang and Xu (2001), but in those works, the parameter is assumed to be constant. Only Kenne, Ahmed-Ali, Lamnabhi-Lagarrigue, and Arzande (2008) consider time-varying parameter.

Numerous approaches were proposed in order to deal with nonlinear system estimation or diagnosis (Alcorta Garcia & Frank, 1997; Carlos-Hernandez et al., 2009). An efficient way consists in rewriting the original nonlinear system in a simpler form, like the Takagi–Sugeno (T–S) model. Originally introduced by Takagi and Sugeno (1985), the T–S representation allows to exactly describe nonlinear systems, under the condition that the nonlinearities are bounded. This is reasonable since state variables as well as parameters of physical systems are bounded (Nagy Kiss, Marx, Mourot, Schultz, & Ragot, 2010; Tanaka & Wang, 2001 and the references therein). The T–S model is a time-varying convex interpolation between linear submodels:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the system state, $u(t) \in \mathbb{R}^{n_u}$ is the control input and $y(t) \in \mathbb{R}^m$ is the system output. $\xi(t) \in \mathbb{R}^q$ is the decision variable assumed to be either measurable (as the system output), known (as the system input) or unmeasured (as the system state). The weighting functions $\mu_i(\xi(t))$ satisfy the convex sum property:

$$\begin{cases} \sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \\ 0 \le \mu_i(\xi(t)) \le 1, \quad i = 1, \ldots, r \end{cases} \tag{2}$$

* Corresponding author at: Université de Lorraine, CRAN, UMR 7039, 2 avenue de la Forêt de Haye, Vandoeuvre-les-Nancy, Cedex 54516, France.
Tel.: +33 383 68 44 19.
E-mail addresses: souad.bezzaoucha@univ-lorraine.fr (S. Bezzaoucha), benoit.marx@univ-lorraine.fr (B. Marx), didier.maquin@univ-lorraine.fr (D. Maquin), jose.ragot@univ-lorraine.fr (J. Ragot).

0967-0661/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.conengprac.2013.06.009

The representation (1), along with the property (2), allows to extend to nonlinear systems the use of some tools developed in the linear framework, for the stability study, the controller design, the observer synthesis and the diagnosis (Nagy Kiss, et al., 2010; Tanaka & Wang, 2001). A systematic and exact transformation of a nonlinear system into a T–S form, without any loss of informations, is known as the Sector Nonlinearity Transformation (SNT) (Nagy, Mourot, Marx, Ragot, & Schutz, 2010; Tanaka & Wang, 2001). Even if SNT leads to T–S models with unmeasurable premise variables, most of the works on T–S systems are devoted to models with known premise variables, since the estimation or diagnosis is obviously easier when the premise variables are accessible. In the following, T–S systems with unmeasurable premise variables (UPM) are studied since they naturally appear when applying the SNT.

In the present paper, a systematic procedure is presented to deal with the state and parameter estimation for nonlinear time-varying systems. It consists in transforming the original system into a T–S system with unmeasurable premise variables using the SNT. Then a joint state and parameter observer is designed for the T–S system with unknown premise variables. Up to the author's knowledge, this is the first contribution where the joint parameter and state estimation problem is addressed in such a way for the nonlinear systems. Moreover, most of the works devoted to joint parameter and state estimation for nonlinear systems only consider constant parameters, whereas time-varying parameters are here studied. The main result is to establish the convergence conditions of the joint state and parameter estimation errors. The observer gains will be derived by solving an LMI optimization problem obtained from the Lyapunov theory. The minimized criterion is the $\mathcal{L}_2$−gain of the transfer from the exogeneous inputs to the state and parameter estimation errors. Using the obtained theoretical results, the joint estimation is performed for a wastewater treatment process (WWTP) modeled by an Activated Sludge Model (ASM1 model) (Weijers, 2000). The data measures used for process simulation are those of the European Program Benchmark Cost 624 (Alex et al., 1999). Indeed, on a practical point of view, it is shown how to estimate the state of the process using the available measurements and how to estimate a parameter which is varying due to external or internal disturbances. The choice of the known inputs, the time-varying parameter (modeling error), the measures and the operating conditions are made by taking into account the specific features of the Bleesbruck treatment station from Luxemburg. The different steps from the process description as a T–S system to the implementation of time-varying parameter and state estimation are clearly detailed.

The paper is organized as follows. Section 2 introduces the T–S representation of the nonlinear time-varying parameter systems. In Section 3, the design of a joint state and parameter observer for T–S system with UPM is presented. Simulation results of the application of the proposed approach to an activated sludge reactor model are given in Section 4. Conclusions are detailed in Section 5.

2. Polytopic modelling of nonlinear time-varying parameter systems

The first contribution of this work is to model nonlinear time-varying systems using the T–S or polytopic representation. For that, each time-varying parameter is rewritten under a particular form.

Let us consider the nonlinear time-varying T–S system represented by Eq. (3) with n time-varying parameters $\theta_j(t)$

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i(\theta(t))x(t) + B_i(\theta(t))u(t)) \\ y(t) = Cx(t) \end{cases} \tag{3}$$

with $\theta(t) = [\theta_1(t) \ \dots \ \theta_n(t)]^T$ and

$$A_i(\theta(t)) = \overline{A}_i + \sum_{j=1}^{n} \theta_j(t)\overline{A}_{ij}$$
$$B_i(\theta(t)) = \overline{B}_i + \sum_{j=1}^{n} \theta_j(t)\overline{B}_{ij} \tag{4}$$

Remark 1. With no loss of generality, it is supposed that the matrices $A_i(\theta(t))$ and $B_i(\theta(t))$ depend on the same time-varying parameters $\theta_j(t)$. If a given matrix A_i (resp. B_i) does not depend on a given $\theta_j(t)$, then $\overline{A}_{ij}$ (resp. $\overline{B}_{ij}$) is null in (4).

For example, if the matrices $A_i(\theta_a(t))$ (resp. $B_i(\theta_b(t))$) depend on $\theta_a \in \mathbb{R}^{n_a}$ (resp. $\theta_b \in \mathbb{R}^{n_b}$), they can be defined as in (4) with $\theta(t) = [\theta_a^T(t), \ \theta_b^T(t)]^T$, $n = n_a + n_b$, $\overline{A}_{ij} = 0$ for $j = n_a + 1, \dots, n$ and $\overline{B}_{ij} = 0$ for $j = 1, \dots, n_a$.

According to the SNT (Tanaka & Wang, 2001), each parameter $\theta_j(t)$ is expressed as a function of its upper and lower bounds, respectively denoted θ_j^1 and θ_j^2 such that

$$\theta_j(t) = \tilde{\mu}_j^1(\theta_j(t))\theta_j^1 + \tilde{\mu}_j^2(\theta_j(t))\theta_j^2 \tag{5}$$

where $\tilde{\mu}_j^1(\theta_j(t))$ and $\tilde{\mu}_j^2(\theta_j(t))$ are defined by

$$\tilde{\mu}_j^1(\theta_j(t)) = \frac{\theta_j(t) - \theta_j^2}{\theta_j^1 - \theta_j^2}$$
$$\tilde{\mu}_j^2(\theta_j(t)) = \frac{\theta_j^1 - \theta_j(t)}{\theta_j^1 - \theta_j^2} \tag{6}$$

and satisfy the convex sum property:

$$\begin{cases} \tilde{\mu}_j^1(\theta_j(t)) + \tilde{\mu}_j^2(\theta_j(t)) = 1, \quad \forall t \\ 0 \le \tilde{\mu}_j^i(\theta_j(t)) \le 1 \end{cases} \tag{7}$$

Replacing (5) in (4), it becomes

$$A_i(\theta(t)) = \overline{A}_i + \sum_{j=1}^{n} \sum_{k=1}^{2} \tilde{\mu}_j^k(\theta_j(t))\theta_j^k \overline{A}_{ij}$$
$$B_i(\theta(t)) = \overline{B}_i + \sum_{j=1}^{n} \sum_{k=1}^{2} \tilde{\mu}_j^k(\theta_j(t))\theta_j^k \overline{B}_{ij} \tag{8}$$

The time-varying matrices $A_i(\theta(t))$ and $B_i(\theta(t))$ can now be written as polytopic matrices. Firstly, due to (7), it follows that

$$\sum_{j=1}^{n} \theta_j(t)\overline{A}_{ij} = \sum_{j=1}^{n} [(\tilde{\mu}_j^1(\theta_j(t))\theta_j^1 + \tilde{\mu}_j^2(\theta_j(t))\theta_j^2)\overline{A}_{ij}]$$
$$= \sum_{j=1}^{n} \left[[(\tilde{\mu}_j^1(\theta_j(t))\theta_j^1 + \tilde{\mu}_j^2(\theta_j(t))\theta_j^2)\overline{A}_{ij}] \times \left[\prod_{k=1, k \ne j}^{n} \sum_{m=1}^{2} \tilde{\mu}_k^m(\theta_k(t)) \right] \right] \tag{9}$$

and thus, Eqs. (8) can be written as

$$A_i(\theta(t)) = \sum_{j=1}^{2^n} \tilde{\mu}_j(\theta(t))\mathcal{A}_{ij}$$
$$B_i(\theta(t)) = \sum_{j=1}^{2^n} \tilde{\mu}_j(\theta(t))\mathcal{B}_{ij} \tag{10}$$

with

$$\begin{cases} \tilde{\mu}_j(\theta(t)) = \prod_{k=1}^{n} \tilde{\mu}_k^{\sigma_j^k}(\theta_k(t)) \\ \mathcal{A}_{ij} = \overline{A}_i + \sum_{k=1}^{n} \theta_k^{\sigma_j^k} \overline{A}_{ik} \\ \mathcal{B}_{ij} = \overline{B}_i + \sum_{k=1}^{n} \theta_k^{\sigma_j^k} \overline{B}_{ik} \end{cases} \tag{11}$$

Then, for n parameters, 2^n submodels are obtained. It is important to note that an analytical expression of the time-varying matrices $A_i(\theta(t))$ and $B_i(\theta(t))$ is obtained with convex weighting functions