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Can an adverse density difference across a surface be stabilized by heating from above?

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ABSTRACT

We investigate the possibility of stabilizing a Rayleigh–Taylor experiment by imposing a small upward temperature gradient. We find that if the two fluids have equal thermal conductivities nothing can be accomplished. If either thermal conductivity is much greater than the other, the small gradient is always stabilizing. If the thermal conductivities are of the same order of magnitude the small gradient can be stabilizing or destabilizing depending on the thermal expansion coefficients.

We have used a Darcy model so that we can derive formulas and present a physical explanation of what we find.

1. Introduction

In the ordinary Rayleigh–Taylor problem, cf., Chandrasekhar [2], we have two fluids of uniform density, a heavy fluid lying above a light fluid. The configuration is gravitationally unstable. A displacement of the surface will lead to its collapse unless the stabilizing effect of surface tension is taken into account. Then the configuration becomes stable to small perturbations of short wavelength, remaining unstable to perturbations of long wavelength. The critical wave number, k, is given by

$$k^2 = \frac{\Delta \rho g}{\gamma}$$

where $\Delta\rho$ denotes the density difference across the surface and γ denotes its surface tension. Thus heavy over light can be maintained in a small diameter container, but not in a large diameter container.

It may be that we wish to observe the instability, possibly to determine the pattern in which the surface breaks. Thus we would like to stabilize the surface, set the experiment up in a large diameter container and then remove the stabilizing forces whereupon the surface becomes unstable. And we might imagine that a small temperature gradient would do the job. Our aim is to do this by heating from above. Alexseev and Oron [1] stabilize the surface by Marangoni convection, Carlés et al. [3] impose a magnetic field and Ratafia [5] accelerates the system.

We impose a temperature gradient on the system, hot above cold, and it might be thought that this would always work. However there is a surprise, hot above cold is not always stabilizing.

To indicate why a positive temperature gradient might be thought to be stabilizing, we present some sketches. The sketch in Fig. 1, drawn at the no-heating critical value of k^2, illustrates a crest that ought to rise and trough that ought to fall if were it not for the stabilizing effect of surface tension.

The sketch in Fig. 2 indicates the effect of a positive temperature gradient imposed on the fluids, cold at the bottom, hot at the top, assuming the same densities obtain as before at the surface.

The buoyancy causing the crest to rise and the trough to fall is less than before, the wave number is the same, so this ought to be a stable picture. And we should conclude that a positive temperature gradient is stabilizing, i.e., that the neutral value of k^2 should decrease on heating.

There is, however, more going on than this simple figure would suggest. For example, offsetting the stabilizing effect of the vertical temperature gradient, is heat conduction from left to right, increasing the densities in the trough, decreasing the densities in the crest, thereby strengthening the adverse buoyancy.

Now these are static pictures. However, at the above no-heating critical point there will be flow if we introduce heating and the flow will affect the pressure difference across the surface; how it does this depends on the thermal conductivities and the coefficients of thermal expansion and this effect may be either stabilizing or destabilizing.

We aim to find out when heating from above is stabilizing and when it is not. We set our problem in a porous solid and use

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Darcy's law to obtain the velocity of the fluid. We do this for algebraic simplicity and to eliminate the effect of surface tension gradient-driven convection. Taking the interface to be of infinite horizontal extent is no limitation in a Darcy fluid model. The side walls simply limit the allowable wave numbers. The greatest loss in using a Darcy model is the loss of viscous coupling. Rasenat et al. [4] account for this via a Navier–Stokes model but, as they say, no formulas can be obtained.

We get our greatest simplification upon observing that what is most important are the pressure and temperature gradients near the surface and not so much what is going on far away. Hence we put the upper and lower boundaries infinitely far away.

Our aim then is to input a small temperature gradient and derive its effect on the critical wave number.

2. The non-linear equations and the base solution

Assuming that only the densities of the fluids depend on temperature and that this dependence is not strong, we write

$$\frac{\mu}{K}\vec{v} = -\nabla p - \rho(T)g\vec{k} \tag{1}$$

and

$$\nabla \cdot \vec{v} = 0 \tag{2}$$

in both phases, where the superscript ★ will distinguish the lighter phase variables.

Denoting the reference density by ρ_0, where ρ_0 is the density of the heavy fluid in the no-heating case, we have

$$\frac{d\rho}{dT} = -\alpha\rho_0 \tag{3}$$

and thus Eqs. (1)–(3) imply

$$\nabla^2 v_z = \frac{K}{\mu}\alpha\rho_0 g\frac{\partial^2 T}{\partial x^2} \tag{4}$$

and

$$\frac{\partial^2 p}{\partial x^2} = \frac{\mu}{K}\frac{\partial v_z}{\partial z} \tag{5}$$

where K denotes the permeability of the solid and α denotes the thermal expansion coefficient of the fluid. The temperature of the fluid satisfies

$$\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T = \kappa\nabla^2 T \tag{6}$$

where κ denotes the thermal diffusivity of the fluid.

We have no side walls, no top and no bottom walls. Our perturbation variables are assumed to be bounded far from the surface separating the two phases.

The surface is denoted $z = Z(x,t)$ and across this surface we have

$$v_z - v_x Z_x = Z_t = v_z^\star - v_x^\star Z_x \tag{7}$$

$$T = T^\star \tag{8}$$

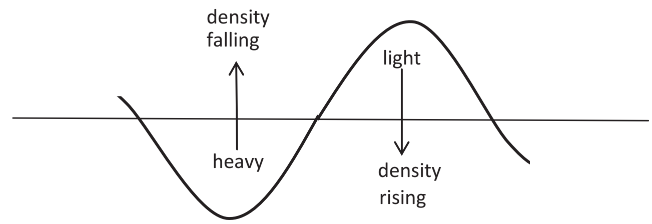

Fig. 2. A displacement at the no-heating critical point in the presence of heating.

$$\lambda\vec{n}\cdot\nabla T = \lambda^\star\vec{n}\cdot\nabla T^\star \tag{9}$$

and

$$p - p^\star = \gamma 2H \tag{10}$$

where λ denotes the thermal conductivity and where the fluids are assumed to be immiscible.

The base solution, due to imposing a small temperature gradient on our two-phase system, is

$$\vec{v}_0 = \vec{0} = \vec{v}_0^\star \tag{11}$$

$$\frac{dp_0}{dz} = -\rho(T_0)g, \quad \frac{dp_0^\star}{dz} = -\rho^\star(T_0)g \tag{12}$$

and

$$\lambda\frac{dT_0}{dz} = \lambda^\star\frac{dT_0^\star}{dz} \tag{13}$$

where the base surface, defining the reference fluid domains, lies at $z = Z_0 = 0$.

The base densities on either side of the surface are denoted ρ_0 and $\rho_0^\star$.

3. The perturbation problem

Imposing a small disturbance on the base solution in the form $\cos kx$, denoting the perturbation variables by the subscript 1 and seeking the critical value of the wave number, k, we write

$$v_{z1} = \hat{v}_{z1}(z)\cos kx$$

$$T_1 = \hat{T}_1(z)\cos kx$$

$$p_1 = \hat{p}_1(z)\cos kx$$

and

$$Z_1 = \hat{Z}_1\cos kx$$

and we have, using Eqs. (4)–(6), for $z \geqslant 0$,

$$\left(\frac{d^2}{dz^2} - k^2\right)\hat{v}_{z1} = -Rk^2\hat{T}_1 \tag{14}$$

$$\hat{v}_{z1}\frac{dT_0}{dz} = \kappa\left(\frac{d^2}{dz^2} - k^2\right)\hat{T}_1 \tag{15}$$

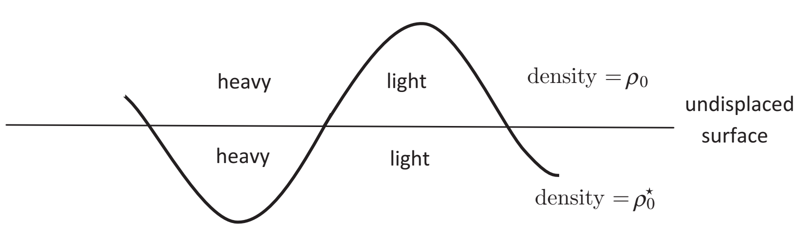

Fig. 1. A displacement at the no-heating critical point.