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Measurement of off-diagonal transport coefficients in two-phase flow in porous media

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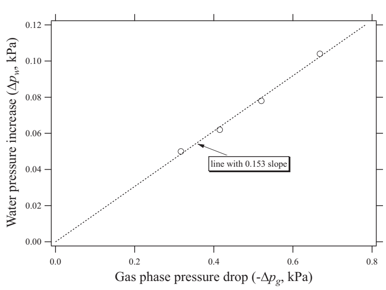
GRAPHICAL ABSTRACT



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ABSTRACT

The prevalent description of low capillary number two-phase flow in porous media relies on the independence of phase transport. An extended Darcy's law with a saturation dependent effective permeability is used for each phase. The driving force for each phase is given by its pressure gradient and the body force. This diagonally dominant form neglects momentum transfer from one phase to the other. Numerical and analytical modeling in regular geometries have however shown that while this approximation is simple and acceptable in some cases, many practical problems require inclusion of momentum transfer across the interface. Its inclusion leads to a generalized form of extended Darcy's law in which both the diagonal relative permeabilities and the off-diagonal terms depend not only on saturation but also on the viscosity ratio. Analogous to application of thermodynamics to dynamical systems, any of the extended forms of Darcy's law assumes quasi-static interfaces of fluids for describing displacement problems.

Despite the importance of the permeability coefficients in oil recovery, soil moisture transport, contaminant removal, etc., direct measurements to infer the magnitude of the off-diagonal coefficients have been lacking. The published data based on cocurrent and countercurrent displacement experiments are necessarily indirect. In this paper, we propose a null experiment to measure the off-diagonal term directly. For a given non-wetting phase pressure-gradient, the null method is based on measuring a counter pressure drop in the wetting phase required to maintain a zero flux. The ratio of the off-diagonal coefficient to the wetting phase diagonal coefficient (relative permeability) may then be determined.

The apparatus is described in detail, along with the results obtained. We demonstrate the validity of the experimental results and conclude the paper by comparing experimental data to numerical simulation.

1. Introduction

In the absence of body forces, based on the early work of Wyckoff and Botset [1], Wyckoff et al. [2], and Leverett [3], the extension of the isotropic form of Darcy's law

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$$\boldsymbol{v} = -\frac{k}{\mu}\nabla p, \tag{1}$$

to multiphase flow is

$$\boldsymbol{v}_i = -\frac{k_{ei}}{\mu_i}\nabla p_i = -\frac{kk_{ri}}{\mu_i}\nabla p_i, \tag{2}$$

where $\boldsymbol{v}$ is the superficial velocity vector, $\boldsymbol{v}_i$ is the superficial velocity for phase i, k is the single phase permeability, k_{ei} and k_{ri} are the saturation dependent effective and relative permeabilities to phase i, μ is the dynamic coefficient of viscosity, and p is the pressure, again, with the subscript i indicating the flowing phase of interest. With $i = n, w$ for non-wetting and wetting phases respectively, the phase pressure difference $p_n - p_w$ is the capillary pressure p_c, whose value determines the fluid saturations S_i in *quasi-static phase replacement* processes. Excluding hysteresis, it is common to assume that k_{ei} is determined by the fluid saturation alone, and its functional dependence on saturation may be obtained by conducting flow experiments that minimally disturb interface equilibrium at the capillary pressure of interest. Application of Eq. (2) to commonly occurring displacement processes in hydrology, and oil and gas industry, requires that over the length scale R needed for defining p_c and k_{ei}, $R\nabla S_i \ll 1$. Furthermore, consistent with the conjecture that each fluid flows in accordance with its own pressure gradient, the effective permeabilities are independent of the viscosity ratio.

In a capillary network, with pore occupancy determined by capillarity, the notion that fluid–fluid interfaces induce only local circulatory motion and do not transfer appreciable macroscopic momentum is the basis of Eq. (2). In the presence of corners and crevices that allow for macroscopically relevant wetting-phase motion, diagonally dominant force-flux relationship is inadequate [4]. To include shear induced motion on the second fluid, Rose [5] hypothesized that an Onsager-type relationship should apply i.e.,

$$\boldsymbol{v}_n = -\lambda_{nn}\nabla p_n - \lambda_{nw}\nabla p_w, \tag{3}$$
$$\boldsymbol{v}_w = -\lambda_{ww}\nabla p_w - \lambda_{wn}\nabla p_n, \tag{4}$$

where it was understood that the mobilities λ_{ij} were the ratio of the effective permeabilities to viscosities; no discussion of how the viscosities appeared in the off-diagonal terms was given by Rose. Invoking the Onsager reciprocity principle, he asserted that $\lambda_{nw} = \lambda_{wn}$. A number of papers have speculated on the notion of a generalized mobility matrix for two-phase flow. For example, deGennes [6] wrote a form identical to Eqs. (3) and (4), with saturation-dependent mobilities. No specific viscosity factor appeared in his formulation.

Bourbiaux and Kalaydjian [7] attempted to measure the off-diagonal coefficients by conducting a cocurrent displacement, followed by a counter-current imbibition. Since the gradients in the two phase-pressures are of opposite sign in counter-current flow, and are more or less matched in co-current flow, the measured profiles or water-cut in the two experiments would be indicative of the magnitude of the coefficients λ_{nw}. Symmetry of the coefficients was assumed for processing the data, and a characteristic peak in λ_{nw} was obtained. Given the viscosity ratio of about ten and the implementation of the boundary conditions, it is difficult to determine whether such an indirect measurement is an artifact or not. For example, no special effort was taken to eliminate capillary end-effects [8,9]. The consequences of end-effects are quite different for counter-current and co-current flow. Furthermore, experiments were performed in natural media (clayey sandstone), and although these were stated to be fairly homogeneous, displacement profiles indicated moderate heterogeneities, the effect of which on the interpretation of data is unclear. Zarcone and Lenormand [10] measured the off-diagonal coefficient by imposing a zero-pressure gradient on water, while pumping mercury through a sand-pack, and found coefficients smaller than 0.01. Similar experiments by Dullien and Dong [11] however resulted in much larger magnitudes than [10]. Dullien and Dong obtained non-symmetric coefficients as well, and were speculating about variability in interface configurations from one experiment to the other (wetting phase gradient as opposed to non-wetting phase gradient). It is clear from their data that there is also a saturation gradient within the sample in these experiments, and the saturation configurations were not the same in the two sets of experiments that they conducted.

Several authors have speculated on the form of the diagonal coefficients, some suggesting that the off-diagonal coefficients are unequal (see e.g. [12]). The symmetry of the coefficients has been demonstrated by Auriault [13], who correctly argues that the off-diagonal coefficients are negligible if the interface is nearly rigid with zero velocities. Others such as Lasseux et al. [14] derive a reciprocal form based on volume averaging and closure conditions through essentially Auriault's [13] method, but were unable to specify a self-consistent order of magnitude for the off-diagonal coefficients. For a viscosity ratio of unity, Rakotomalala et al. [15], computed off-diagonal coefficients that were about 1% compared to the diagonal ones.

The network modeling of Goode and Ramakrishnan [4] relied first on a finite element analysis within a single pore to compute a duct conductance matrix, based on which it was shown that the off-diagonal coefficients were *equal*.[2] However both duct conductance matrix coefficients and the effective network permeability matrix were seen to depend on both the saturation and the viscosity ratio of the flowing fluids. The coefficients were computed with stationary interfaces, and therefore a pressure perturbation was imposed on one of the fluids to compute the matrix of mobility coefficients. The general conclusion may be written compactly for quasi-static two-phase flow as

$$\boldsymbol{v}_w = -\frac{kk_{rw}(S_w; M)}{\mu_w}\nabla p_w - \frac{kk_{rc}(S_w; M)}{\mu_w + \mu_n}\nabla p_n, \tag{5}$$
$$\boldsymbol{v}_n = -\frac{kk_{rc}(S_w; M)}{\mu_w + \mu_n}\nabla p_w - \frac{kk_{rn}(S_w; M)}{\mu_n}\nabla p_n, \tag{6}$$

where k_{rc} is the off-diagonal or cross-coefficient, k_{rw} and k_{rn} are the wetting and the non wetting phase relative permeabilities, and M is the viscosity ratio equal to μ_n/μ_w. Note that in our formulation, the relative permeabilities and the cross-coefficients are functions of saturation and viscosity ratio, and is different from those proposed by Rose [5] and deGennes [6], where only explicit dependence on saturation alone is assumed. The dependence of the wetting fluid relative permeability (k_{rw}) on M is however rather weak and may be ignored for practical purposes. The key conclusion of Goode and Ramakrishnan [4] was that a nonzero k_{rc} implies a dependence on M for all of the three coefficients, whereas the converse is not *necessarily* true. For example, when the wetting phase is disconnected, k_{rn} may depend on M, but k_{rc} may be negligible since the motion near the interfaces do not appreciably contribute to the superficial velocity. Also, the form chosen in [4] for the viscosity dependence of the off-diagonal mobility is also symmetric. This form also gives comparable magnitudes for k_{rc} with respect to the viscosity ratio.

Clearly, the experimental results have varied, and while some inferences have been indirect, others have used methods where the velocities are measured, while maintaining a zero pressure gradient in one of the phases. The latter have been performed in sand-packs and consistent results with regard to the

[2] Contrary to the statements given by Li et al. [16] that [4] implied invalidity of reciprocity.