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Subspace identification analysis of RFX and T2R reversed-field pinches

K. Erik J. Olofsson [a,*], Anton Soppelsa [b], Tommaso Bolzonella [b], Giuseppe Marchiori [b]

[a] EES School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden
[b] Istituto Gas Ionizzati (IGI), Consorzio RFX, Corso Stati Uniti 4, 35127 Padova, Italy

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ABSTRACT

Input–output datasets from two magnetic confinement fusion (MCF) experiments of the reversed-field pinch (RFP) type are examined. The RFP datasets, which are samples of the distributed magnetic field dynamics, are naturally divided into many smaller batches due to the pulsed-plasma operation of the experiments. The two RFP experiments considered are (i) EXTRAP T2R (T2R) with 64 inputs and 64 outputs and (ii) RFX-mod (RFX) with 192 inputs and 192 outputs. Both T2R and RFX are magnetohydrodynamically unstable and operates under magnetic feedback with optional dither injection. Using subspace system identification techniques and randomised cross-validation (CV) methods to minimise the generalisation error, state-space orders of the empirical systems are suggested. These system orders are compared to "stabilisation diagrams" commonly used in experimental modal analysis practice. The relation of the CV system order to the decay of the singular values from the subspace method is observed. Both (i) stable vacuum diffusion and (ii) unstable plasma response datasets are analysed. Apparent simulation and prediction errors are quantified for both cases using a deviation-accounted-for index. These results are purely data-driven. A simple approach towards exploitation of the subspace techniques for finite-element model refinement and data confrontation is presented.

1. Introduction

Resistive wall modes (RWMs) constitute a particular class of magnetohydrodynamic (MHD) instabilities that is believed to set a limit to the efficiency of power generation from future fusion reactors of the tokamak type (Chu & Okabayashi, 2010). Tokamaks (Wesson, 2004) are the most developed reactor prototype in the magnetic confinement approach to fusion (MCF). The experimental study of MHD instabilities and marginal modes is of crucial importance for the development of MCF designs in general and for tokamaks in particular. MHD is a magnetofluid theory that models the macroscopic and to some extent microscopic behaviour of e.g. current-carrying hot MCF plasmas (Freidberg, 1987; Stacey, 2005). To achieve thermonuclear fusion the plasma (a deuterium–tritium mixture) needs to be heated to about ~100 million Kelvins. It turns out that when the thermal-kinetic pressure in MHD is increased up to the proximity to a critical level in relation to the confining magnetic field, the RWM class modes may grow and disrupt (terminate) the plasma. When this happens the reactor may stop abruptly and possibly expensive damage to the machine may be inflicted due to large induced currents and forces, and localised heat deposition onto the plasma facing surfaces. This would not be tolerated in a continuously operating power plant.

RWMs is one type of MHD instability that can be studied in another MCF device called a reversed-field pinch (RFP) (Bodin & Newton, 1980; Chu & Okabayashi, 2010; Gimblett, 1986; Martin, 2011). RFPs and tokamaks are similar in many respects and it is sometimes argued that it is possible to do tokamak reactor relevant experiments with RFPs. In particular the RFP has experimentally demonstrated the principle of reliable RWM stabilisation using arrays of current-carrying feedback coils. This was first achieved with EXTRAP T2R (T2R, Brunsell et al., 2001) and then reproduced with the larger RFX-mod (RFX, Chitarin, Bello, Grando, & Peruzzo, 2003; Sonato et al., 2003) device.[1]

It would seem that closed-loop system identification techniques (Söderström & Stoica, 1989; Zhu, 2001) could enable "plant-friendly" experimental modal analysis (EMA) campaigns of e.g. RWMs in RFPs. This was recently achieved at T2R by developing specific multibatch subspace system identification methods (SIMs, overviewed in e.g. Qin, 2006) as detailed in Olofsson, Rojas, Hjalmarsson, Brunsell, and Drake (2011), Olofsson, Brunsell, and Drake (2012), and Olofsson (2012). No tokamak has so far reported a comparable study but several tokamaks are being retrofitted with coil systems that may allow such techniques to be implemented and tested in the future.

This paper focuses on two unique (first of its kind) datasets recently obtained from closed-loop stabilised operation with the

* Corresponding author. Tel.: +46 739460154.
E-mail addresses: erik.olofsson@ee.kth.se, ko2319@columbia.edu (K.E.J. Olofsson).

[1] To be clear: throughout this paper, the abbreviation "RFX" will always refer to the re-engineered and updated machine "RFX-mod" (Sonato et al., 2003).

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above mentioned RFP experiments: RFX and T2R.[2] The specific issues addressed are model order selection for SIMs and the relation to stabilisation diagrams commonly inspected in EMA. Empirical results for Monte-Carlo cross-validation (MCCV) and Efron's .632-bootstrap (Hastie, Tibshirani, & Friedman, 2009) for estimation of the SIM generalisation error (supervised learning) are presented. These results are compared with the canonical-correlation-weighted singular values of the SIM and the stabilisation diagram (unsupervised learning) for the convergence of eigenvalues.

There are many envisionable applications of the techniques presented herein. Examples may include (i) fault-detection and isolation signal processing that monitors MHD and coil-system behaviour in real-time, (ii) adaptive MHD control using online SIMs as an MHD-spectroscopic tool, (iii) alternative means to benchmark and adjust finite-element (FE) models and suggest FE order complexity. The third example alludes to the disconnect (unavoidable discrepancies) between nominal FE models and linear MHD stability, and the "noisy" properties (fluctuating and stochastic signals, non-modelled dynamics, neglected dynamics, uncertain dynamics) of real MCF systems. An example of three-dimensional eddy current models coupled to linear MHD for RFPs can be found in reference (Villone, Liu, Paccagnella, Bolzonella, & Rubinacci, 2008).

The paper is organised as follows. Section 2 introduces the RFPs analysed in this study. In this section both control signal schematics and gross machine parameters are presented. Section 3 outlines the SIM employed for analysis. Sections 4 reports the evaluation of system eigenvalues and covariance singular values for the vacuum and plasma datasets. Section 5 reports the corresponding cross-validated state-space system orders. Section 6 considers the problem of confronting prior FE-models with data. Conclusions end the paper in Section 7.

2. EXTRAP T2R and RFX-mod reversed-field pinches

The RFPs T2R and RFX are two different RFP experiments. RFX has recently demonstrated sustained plasma current I_{pl} up to 2 MAs. T2R can operate a little above $I_{pl}\approx 100$ kA. Both RFX and T2R are equipped with extensive saddle coil arrays used for magnetic diagnostics and feedback stabilisation. The RFX setup of 48 toroidal locations each with 4 poloidal coils (from here-on denoted 4×48) is depicted in Fig. 1. T2R has an analogous configuration of 4×32 used for feedback.[3] An example evolution of the plasma current in a single RFX experiment (in the present campaign) is plotted in the top panel of Fig. 3.

The array of coils utilised in RFX and T2R basically implements the idea first published by Bishop (1989). Theoretical pre-studies of RFP-specific feedback using coil arrays include (Fitzpatrick & Yu, 1999; Paccagnella, Gregoratto, & Bondeson, 2002). The first affirmative experimental results on the feasibility of feedback stabilisation of resistive wall MHD in RFPs were obtained at T2R (Brunsell, Yadikin, Gregoratto, Paccagnella, & Bolzonella, 2004). RFX further proved successful stabilisation of RWMs at higher plasma current level (Ortolani & RFX team, 2006). Moreover, mitigation of plasma surface deformations induced by wall-locked tearing modes was obtained thanks to real-time correction of a systematic error in Bishop's intelligent-shell approach (Zanca, Marrelli, Manduchi, & Marchiori, 2007). Important RFX control engineering developments include (Marchiori & Soppelsa, 2007; Marchiori et al., 2012; Soppelsa, Marchiori, Marrelli, & Villone, 2009). Recent studies of RFP RWM stability and feedback physics and modelling include (Bolzonella et al., 2008) and the work of Wang with co-authors (Wang & Guo, 2011; Wang et al., 2010).

The plant G in the schematics of Fig. 2 is the main object of interest. The input vector $\mathbf{u}_G$ to G is the currents that are driven in the red saddle coils outermost in Fig. 1. These coils are known as the "actuators". The output vector of G, the signal $\mathbf{y}$, are the time-integrated voltages resulting from the change of magnetic flux through the blue innermost saddle coils in Fig. 1. Thus the system G is supposed to describe the dynamics of the distributed magnetic field in the particular geometry shown, with the required eddy current evolution of, and coupling to, the passive electrically conducting copper shell (also depicted in Fig. 1). If there is no RFP plasma in the machine then this evolution is referred to as a vacuum diffusion system. The intended operation of plant G, with RFP plasma present, results in an unstable magnetic field evolution. The RFP plasma will also imply a higher process noise than for the vacuum system. The signal $\mathbf{v}$ in Fig. 2 denotes both the unmeasured disturbances of the system G and its output error noise.

For the present purposes the time-scale ratios given in Table 1 are of particular relevance. The geometrical ratios and the total plasma currents are also given in the table. R denotes the torus major radius and a denotes the torus minor radius. The parameters b and c are minor radii giving the locations of saddle loops (as illustrated in Fig. 1). The sampling time-interval τ_s are those used during both acquisition and control of the experiments analysed in this work. It should be understood that the analysis and control time-scales here reported do not cover all the important MHD physics involved in the systems. In particular, only MHD modes that are (nearly and totally) locked to the resistive wall in the laboratory frame are believed to be reliably detected using the outputs $\mathbf{y}$. The two main classes of these modes are RWMs and tearing-modes (TMs). Mainly "saturated-regime" TM activity is likely to influence this time-scale. See further Section 4.3. Any deeper discussion of MHD physics issues are beyond the scope of this paper.

Both RFX and T2R are computer controlled. The signal schematics of Fig. 2 show the digital inputs $\mathbf{r}$ and $\mathbf{d}$ for the respective plants. F is the power amplifier (PA) system that drive currents in the active coils.[4] K denotes the current-controllers of RFX-mod. For T2R it seems sufficient to ignore the potential issues from the error in the measurement of the active coil currents denoted by $\mathbf{u}_G$ in both Fig. 2(a) and (b). This seems not to be the case for RFX presently. The problem of mismeasured system inputs is called errors-in-variables (EIV) in the statistical literature (Söderström, 2007). See further Section 3.1.

2.1. Experimentally acquired datasets

Table 2 lists the types and sizes of the measured system data from the campaigns at T2R and RFX relevant for this study. A set of experiments, denoted by $\mathcal{S}_{(\cdot)}^{(\cdot)}$, is a collection of disjoint experimental time-series recordings, each of approximately equal length. A single experiment is called a "shot". The experimental data of a shot is usually called a "batch". The experimental conditions within each set should be comparable. Two types of sets are considered: (i) open-loop operated vacuum diffusion system experiments $\mathcal{S}_{(\cdot)}^{vac}$ and (ii) closed-loop stabilised plasma response experiments $\mathcal{S}_{(\cdot)}^{pla}$. Open-loop means that the system output $\mathbf{y}$ is not reconnected back to the input $\mathbf{r}$ for RFX (or $\mathbf{d}$ for T2R). See

[2] The T2R closed-loop dither injected campaigns were mainly conducted 2009–2010, with the first trials late 2008. The RFX ditto late 2011 and early 2012.

[3] The T2R saddle coil sensor array is actually 4×64, whereas the active coil array is 4×32 (wide coils so the surface coverage is nearly 100%). Currently a "pairwise-difference" connection is used for both sensor and actuators at 32 toroidal locations to effectively reduce the array to 2×32 for real-time feedback (one vertical component and one horizontal component of the magnetic radial field at each toroidal segment).

[4] RFX PAs are of IGBT type (Toigo, Gaio, Balbo, & Tescari, 2003). T2R PAs are audio-grade ~1 kW pieces of equipment.