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Integrating dynamic economic optimization and model predictive control for optimal operation of nonlinear process systems

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ABSTRACT

In this work, we propose a conceptual framework for integrating dynamic economic optimization and model predictive control (MPC) for optimal operation of nonlinear process systems. First, we introduce the proposed two-layer integrated framework. The upper layer, consisting of an economic MPC (EMPC) system that receives state feedback and time-dependent economic information, computes economically optimal time-varying operating trajectories for the process by optimizing a time-dependent economic cost function over a finite prediction horizon subject to a nonlinear dynamic process model. The lower feedback control layer may utilize conventional MPC schemes or even classical control to compute feedback control actions that force the process state to track the time-varying operating trajectories computed by the upper layer EMPC. Such a framework takes advantage of the EMPC ability to compute optimal process time-varying operating policies using a dynamic process model instead of a steady-state model, and the incorporation of suitable constraints on the EMPC allows calculating operating process state trajectories that can be tracked by the control layer. Second, we prove practical closed-loop stability including an explicit characterization of the closed-loop stability region. Finally, we demonstrate through extensive simulations using a chemical process model that the proposed framework can both (1) achieve stability and (2) lead to improved economic closed-loop performance compared to real-time optimization (RTO) systems using steady-state models.

1. Introduction

Economic optimization of chemical processes has traditionally been addressed through a two-layered architecture. In the upper layer, real-time optimization (RTO) carries out economic process optimization by computing optimal process operation set-points using steady-state process models. These set-points are used by the feedback control systems in the lower layer, typically designed via model predictive control (MPC) methods, to force the process to operate on these steady-states (Backx, Bosgra, & Marquardt, 2000; Marlin & Hrymak, 1997). MPC has been widely adopted in the chemical process industry because of its ability to optimally control multiple-input multiple-output nonlinear systems by solving an on-line optimization problem subject to input and state constraints (García, Prett, & Morari, 1989; Mayne, Rawlings, Rao, & Scokaert, 2000) and minimizes a typically quadratic performance index along a finite prediction horizon. The main disadvantage of this traditional two-layer approach to economic process optimization with RTO and MPC is that RTO does not account for process dynamics or guarantee that the computed set-points are reachable (Rawlings, Bonné, Jørgensen, Venkat, & Jørgensen, 2008). In recent years, numerous calls for the development of the so-called "smart manufacturing paradigm" have led to several attempts to integrate MPC and economic optimization of chemical processes to deal with variable demand, changing energy prices, variable feedstock, and product transitions (Adetola & Guay, 2010; Backx et al., 2000; Tvrzská de Gouvêa & Odloak, 1998; Engell, 2007; Kadam & Marquardt, 2007; Rawlings & Amrit, 2009; Zanin, Tvrzská de Gouvêa, & Odloak, 2002).

Early attempts on integrating MPC and economic optimization have primarily focused on two strategies: (1) integrating steady-state optimization directly in the MPC as in Tvrzská de Gouvêa and Odloak (1998), Zanin et al. (2002), and Yousfi and Tournier (1991) and (2) a two-layer approach similar to traditional control architectures with RTO and MPC that incorporates a dynamic process model in place of a steady-state model in the upper layer called dynamic real-time optimization (D-RTO) (Kadam & Marquardt, 2007; Kadam et al., 2003; Würth, Hannemann, & Marquardt, 2009, 2011; Würth, Rawlings, & Marquardt, 2009; Zhu, Hong, & Wang, 2004). In recent work, the MPC has been

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extended to solve optimization problems with general economic cost functions replacing the convectional quadratic cost of the standard MPC. This combines dynamic economic process optimization and feedback control into one layer. Several economic MPC (EMPC) schemes have been proposed (see Amrit, Rawlings, & Angeli, 2011; Chen, Heidarinejad, Liu, & Christofides, 2012; Diehl, Amrit, & Rawlings, 2011; Heidarinejad, Liu, & Christofides, 2012a, 2012b; Hovgaard, Larsen, Edlund, & Jørgensen, 2012; Huang, Harinath, & Biegler, 2011; Ma, Qin, Salsbury, & Xu, 2012; Rawlings & Amrit, 2009 and the references therein). In Heidarinejad et al. (2012a), general methods were proposed to design an EMPC using Lyapunov-based techniques capable of optimizing closed-loop performance with respect to general economic considerations for nonlinear systems. Moreover, this approach allows for an explicit characterization of the set of initial conditions whereby closed-loop stability and feasibility of the EMPC optimization problem are guaranteed.

While the proposed EMPC approaches have demonstrated closed-loop economic performance improvement, these approaches treat dynamic economic process optimization and control in a one layer approach. This shift from the traditional two layer control paradigm to a one layer framework requires a complete redesign of the existing control architectures. Additionally, considering that EMPC must use a sufficiently large prediction horizon to adequately account for a time-varying economic cost, the EMPC optimization problem may not be solved fast enough to control a process in real-time. While many D-RTO structures have been proposed throughout the literature (for example, Kadam & Marquardt, 2007; Würth et al., 2011; Zhu et al., 2004), many of the two-layered D-RTO and MPC systems proposed are characterized by a lack of rigorous theoretical treatment including the constraints required on the upper level dynamic economic optimization problem to guarantee that the computed optimal time-varying reference state trajectories can be tracked by the lower process control layer as well as an explicit characterization of the set of initial conditions whereby closed-loop stability and feasibility are guaranteed in the lower layer.

Accounting for these considerations, we design, in the present work, a two-layered dynamic economic optimization and control framework. In the upper layer, an EMPC is designed to compute economically optimal time-varying state trajectories in an on-line fashion using real-time measurements. In the lower layer, a LMPC system is used to force the system to track the economically optimal state trajectories taking advantage of its stability and robustness properties (see Christofides & El-Farra, 2005; Mhaskar, El-Farra, & Christofides, 2005, 2006; Muñoz de la Peña & Christofides, 2008). Lyapunov techniques are used to characterize, a priori, the set of initial conditions starting from where feasibility and closed-loop stability are guaranteed. Through rigorous theoretical treatment, we prove practical closed-loop stability of the proposed integrated dynamic economic optimization and control framework. We demonstrate through extensive simulations using a CSTR chemical process model with a time-dependent economic cost function that such an integrated control paradigm can both (1) render the closed-loop time-varying state evolution in a bounded region and (2) perform economically better than traditional RTO systems using steady-state models.

2. Preliminaries

2.1. Notation

The operator $|\cdot|$ is used to denote the Euclidean norm of a vector and $|\cdot|_Q$ denotes the weighted Euclidean norm of a vector (i.e., $|x|_Q = x^T Q x$). A continuous function $\alpha : [0,a) \to [0,\infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$. We use $\Omega_{\rho(x_E)}$ to denote the set $\Omega_{\rho(x_E)} := \{e \in \mathbf{R}^{n_x} \,|\, V(e,x_E) \le \rho(x_E)\}$ for a fixed $x_E \in \Gamma$. The symbol diag(ν) denotes a square diagonal matrix with diagonal elements equal to the vector ν and the symbol

$$\operatorname*{proj}_{\Gamma}(x)$$

denotes the projection of x onto the set Γ.

2.2. Class of process models

In this work, we consider the class of nonlinear systems described by the following state-space model:

$$\dot{x}(t) = f(x(t),u(t),w(t)) \tag{1}$$

where $x(t) \in \mathbf{R}^{n_x}$ is the state vector, $u(t) \in U \subset \mathbf{R}^{n_u}$ is the manipulated input vector, $w(t) \in \mathbf{R}^{n_w}$ is the disturbance vector. The inputs are restricted to be in a nonempty convex set defined as $U := \{u \in \mathbf{R}^{n_u} \,|\, |u_i| \le u_i^{\max},\ i = 1,\dots,n_u\}$. We assume that f is locally Lipschitz on $\mathbf{R}^{n_x} \times \mathbf{R}^{n_u} \times \mathbf{R}^{n_w}$ and the disturbance vector is bounded

$$|w(t)| \le \theta \tag{2}$$

where $\theta > 0$.

We propose a dynamic economic optimization and control framework to force the system of Eq. (1) to track slowly time-varying operating policies. The slowly time-varying trajectory vector is denoted as $x_E(t) \in \Gamma \subset \mathbf{R}^{n_x}$, where Γ is a compact (closed and bounded) set and the rate of change of the reference trajectory is bounded by

$$|\dot{x}_E(t)| \le \gamma_E \tag{3}$$

We define the deviation between the actual state trajectory $x(t)$ and the slowly-varying reference trajectory $x_E(t)$ as

$$e(t) = x(t) - x_E(t) \tag{4}$$

with its dynamics described by

$$\begin{aligned}\dot{e}(t) &= f(x(t),u(t),w(t)) - \dot{x}_E(t)\\ &= f(e(t)+x_E(t),u(t),w(t)) - \dot{x}_E(t)\\ &:= g(e(t),x_E(t),\dot{x}_E(t),u(t),w(t))\end{aligned} \tag{5}$$

We assume that the system of Eq. (5) has a continuously differentiable, isolated equilibrium for each fixed $x_E \in \Gamma$ (i.e., there exists a u_E for a fixed x_E to make $e = 0$ the equilibrium of Eq. (5))

$$g(0,x_E,0,u_E,0) = 0 \tag{6}$$

Remark 1. The assumption that the system of Eq. (1) has an equilibrium for every fixed $x_E \in \Gamma$ is a necessary assumption to guarantee that the reference trajectory can be tracked. While this assumption does require the system to have enough degrees of freedom (e.g., one manipulated input for each time-varying state to track), a system with many states most likely will not include all states in the economic cost. In this case, only a few states would be forced to track reference trajectories. If we remove this assumption and the system is driven away from perfectly tracking the slowly-varying trajectory $x_E(t)$, due to a disturbance for example, no guarantee can be made that the system will ever be driven back to the slowly-varying reference trajectory.

2.3. Stability assumption

We need to make certain assumptions about the system of Eq. (5) to guarantee that the slowly-varying state trajectory $x_E(t)$ can be tracked. For each fixed $x_E \in \Gamma$, we assume that there exists a Lyapunov-based controller $h(e(t),x_E)$ that makes the origin $e = 0$ of the nonlinear system given by Eq. (5) without uncertainty ($w(t) \equiv 0$) asymptotically stable under continuous implementation.