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Event based sampling in non-linear filtering ☆

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ABSTRACT

Most of the existing approaches to estimation and control are based on the premise that regular sampling is used. However, in some applications, there exists strong motivation to use "event" rather than "time" based sampling. For example, in sensor networks, it is often desirable to send data only when "something interesting" happens. This paper explores some of the issues involved in event based sampling in the context of non-linear filtering. Several examples are presented to illustrate the ideas.

1. Introduction

Most current implementations of digital control and estimation use regular sampling with fixed period *T*, see e.g. Middleton and Goodwin (1990), Feuer and Goodwin (1996), Åström and Wittenmark (1990) and Hristu-Varsakelis and Levine (2005). However, there is often strong practical motivation to change this paradigm to one in which one only takes samples "when something interesting" happens. This changes the focus to, so-called, "event based" sampling. In this paper, we consider that a measurement is sent only when the measured variable crosses a given threshold. Thus the sampling is not regular. The latter strategy has many advantages including conserving valuable communication resources in the context of networked control or sensor networks.

There is a growing literature on event based sampling. An early seminal paper was that of Åström and Bernhardsson (2002). Other related publications include Årzén (1999), Anta and Tabuada (2009, 2008), Byrnes and Isidori (1989), Otanez, Moyne, and Tilbury (2002), Tabuada (2007), Le and McCann (2007), McCann and Le (2008), Pawlowski et al. (2009), and Xu and Cao (2011). As pointed out in Anta and Tabuada (2010), event based sampling and control are particularly attractive for non-linear systems since the nature of the system response can be operating point dependent and this may mean that different sampling strategies are desirable at different operating points.

The current paper examines some of the issues related to event based sampling for non-linear filtering. An event based non-linear filter is developed. It is also shown, how such a filter can be implemented using approximate non-linear filtering algorithms including particle filtering (Chen, 2003; Handschin & Mayne, 1969; Schön, 2006) and minimum distortion filters (Cea, Goodwin, & Feuer, 2010; Goodwin, Feuer, & Müller, 2010).

One issue that needs careful consideration in the context of event based filtering is that of the anti-aliasing filter. It is argued here that an alternative viewpoint needs to be adopted for the design of this filter.

The layout of the remainder of this paper is as follows: Section 2 reviews continuous time stochastic models. Section 3 describes basic sampling strategies. Section 4 describes the core ideas behind regular and event based sampling. Section 5 describes sampled data models. Section 6 reviews the traditional discrete non-linear filter. Section 7 details modifications that are required in the discrete non-linear filter to incorporate event based sampling. Section 8 briefly describes approximate discrete non-linear filters. Section 9 presents a realistic example. Section 10 draws conclusions.

2. A continuous time non-linear model

Most physical systems evolve in continuous time and are hence described by ordinary differential equations. A stochastic version of such equations takes the following conceptual form:

$$\frac{d\mathbf{x}}{dt} = f_c(\mathbf{x}) + g_c(\mathbf{x})\frac{d\omega}{dt} \tag{1}$$

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$$\frac{d\mathbf{z}}{dt} = h_c(\mathbf{x}) + \frac{d\mathbf{v}}{dt} \tag{2}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector and $d\mathbf{z}/dt \in \mathbb{R}^m$ is the measured output vector. In (1) and (2), $d\boldsymbol{\omega}/dt$, $d\mathbf{v}/dt$ represent independent continuous time "white noise" processes of intensity $\mathbf{Q}_c$ and $\mathbf{R}_c$ respectively. An important observation is that continuous time white noise does not exist in any meaningful sense. (For example, if one calculates the auto-covariance of such a process, then it takes the form $\mathbf{Q}_c\delta(t)$ where $\delta(\cdot)$ is the dirac delta function.) To overcome this difficulty, it is often more insightful to use spectral density description for the noise. Spectral density is the Fourier transform of the autocorrelation i.e.

$$\left(\text{Spectral density of } \frac{d\boldsymbol{\omega}}{dt}\right) = \int_{-\infty}^{\infty} \mathbf{Q}_c\delta(t)e^{-j\omega t}\,dt = \mathbf{Q}_c \tag{3}$$

Thus $\mathbf{Q}_c$ is the spectral density of the process $\{d\boldsymbol{\omega}/dt\}$. White noise has constraint spectral density over an infinite bandwidth. This observation allows one to supplement the notion of "white noise" by the notion of "broad band" noise which has constant spectrum over a wide (but not infinite) bandwidth. Indeed, it turns out that "whiteness" of the process and measurement noise is largely irrelevant to the operation of an optimal filter. What is actually needed is that the spectrum be substantially constant in key regions. This issue is discussed in detail in Goodwin, Agüero, Salgado, and Yuz (2009). These ideas expose a difficulty with the common practice of using variances to describe noise in the discrete time case. For example, say that the noise is broadband (but non-white) having spectral density $\mathbf{Q}$ covering a bandwidth of W, then the associated variance $\mathbf{V}$ is equal to the area under the spectrum, i.e. $\mathbf{V} = W\mathbf{Q}$. If one uses spectral density to describe the noise, then no difficulties will be encountered since the noise intensity has been correctly captured. However, say that the Nyquist frequency, $1/(2\Delta)$, is greater than the noise bandwidth. Then, if one uses variance to describe the associated filter, then the variance must be artificially scaled to $\overline{\mathbf{V}} = \mathbf{V}/W\Delta$ to match the spectral densities. If this is not done then the associated filter will perform badly due to underestimation of the noise intensity.

A related problem is that variance does not indicate the difficulty of an estimation problem. For example, consider the case of very fast sampling. Then $1/\Delta$ will be large. In this case, a small noise intensity i.e. small spectral density could be associated with a large noise variance. Yet, most of this noise power will lie at frequencies above the bandwidth of the system. Intuitively this part of the noise will not effect the filter performance. Again, it is only the spectral density in relevant parts of the spectrum that effects filter performance.

The above difficulties are overcome if one works with spectral density rather than variance. Moreover, this aligns the continuous and discrete cases, since spectral density (or equivalently incremental variance) is exclusively used in the continuous case.

In view of the above discussion, Eqs. (1) and (2) are more appropriately expressed in incremental form as:

$$d\mathbf{x} = f_c(\mathbf{x})\,dt + g_c(\mathbf{x})\,d\boldsymbol{\omega} \tag{4}$$

$$d\mathbf{z} = h_c(\mathbf{x})\,dt + d\mathbf{v} \tag{5}$$

where the processes $\boldsymbol{\omega}$ and $\mathbf{v}$ correspond to Brownian motion process having incremental covariance $\mathbf{Q}_c\,dt$ and $\mathbf{R}_c\,dt$ respectively. Also, as discussed above, $\mathbf{Q}_c$ and $\mathbf{R}_c$ can equivalently be thought of as spectral densities for $d\boldsymbol{\omega}/dt$ and $d\mathbf{v}/dt$ respectively.

The linear equivalents of Eqs. (4) and (5) are

$$d\mathbf{x} = \mathbf{A}_c x\,dt + d\boldsymbol{\omega} \tag{6}$$

$$d\mathbf{z} = \mathbf{C}_c x\,dt + d\mathbf{v} \tag{7}$$

$\mathbf{x} \in \mathbb{R}^n$, $\mathbf{z} \in \mathbb{R}^m$, $\mathbf{A}_c \in \mathbb{R}^{n\times n}$, $\mathbf{C}_c \in \mathbb{R}^{m\times n}$, $d\boldsymbol{\omega} \in \mathbb{R}^n$ and $d\mathbf{v} \in \mathbb{R}^m$ are the state, measured output, system matrices, process noise and measurement noise respectively. The initial state satisfies $E\{\overline{\mathbf{x}}_0\} = \hat{\mathbf{x}}_0$ and $E\{(\overline{\mathbf{x}}_0 - \hat{\mathbf{x}}_0)^T(\overline{\mathbf{x}}_0 - \hat{\mathbf{x}}_0)\} = \hat{\mathbf{P}}_0$. In the linear case, $\boldsymbol{\omega}$ and $\mathbf{v}$ are assumed to be stationary vector Wiener processes with incremental covariance $\mathbf{Q}_c\,dt$ and $\mathbf{R}_c\,dt$ respectively. The matrices $\mathbf{Q}_c$ and $\mathbf{P}_0$ are assumed to be symmetric and positive semidefinite, and $\mathbf{R}_c$ is assumed to be symmetric and positive definite.

3. Choice of sampling strategy

Consider first the case of regular sampling with fixed period Δ. (This is sometimes called Riemann sampling (Åström & Bernhardsson, 2002). Here the focus is on the independent time variable).

In Section 2, $d\mathbf{z}/dt$ was defined as the continuous time output (see Eqs. (2), (5) and (7)). The next step is to develop the form of the model when samples are taken. However, this begs the question, "Samples of what?". Two possible options are explored below for the *sampled output*.

3.1. Direct sampling of $d\mathbf{z}/dt$

At first glance, it seems plausible that one could directly sample the continuous process $d\mathbf{z}/dt$. However, this choice is actually an infeasible option since the samples of the associated noise, $d\mathbf{v}/dt$, would have infinite variance!

3.2. Sampling after passing through an anti-aliasing filter

An appropriate remedy to the difficulty described in Section 3.1 is to pass $d\mathbf{z}/dt$ through an anti-aliasing filter prior to sampling. A common choice for such a filter is to simply average $d\mathbf{z}/dt$ over the sample period. Actually, some form of averaging is inherent in all low pass filters that are typically used as anti-aliasing filters. In the case of averaging, the sampled output satisfies:

$$\overline{\mathbf{y}}_k = \frac{1}{\Delta}\int_{k\Delta}^{(k+1)\Delta} \frac{d\mathbf{z}}{dt} \tag{8}$$

$$\overline{\mathbf{y}}_k = \frac{1}{\Delta}\{\mathbf{z}((k+1)\Delta) - \mathbf{z}(k\Delta)\} \tag{9}$$

To obtain a notation for the sampled data case which resembles the continuous case, the (discrete) increment in z is defined via

$$d\mathbf{z}^+ = \mathbf{z}((k+1)\Delta) - \mathbf{z}(k\Delta) \tag{10}$$

where the superscript '+' denotes "next" sampled value. In this case, Eq. (9) can be rewritten as

$$\overline{\mathbf{y}}_k = \frac{1}{\Delta}\,d\mathbf{z}^+ \tag{11}$$

4. Event based sampling

Next consider the case of event based sampling. (This is sometimes called Lebesgue sampling (Åström & Bernhardsson, 2002). Here the focus is on the dependent variable).

Let $\{q_{ij}\}$ be a set of quantization levels for the jth output. These quantization levels could, for example, be evenly spaced so that

$$q_{i+1,j} - q_{i,j} = L_j \in \mathbb{R} \quad \text{for } j = 1,\ldots,n \tag{12}$$

In event based sampling, the measured output is transmitted only when a quantization level has been crossed. Moreover, provided no bits are lost and provided a starting signal level is known, then only 1 bit/sample needs to be sent to indicate that the signal has moved to the next interval above $(+1)$ or the next interval below (-1). The difference between Riemann and Lebesgue sampling is illustrated in Fig. 1.