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# An application of system identification in metrology

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ABSTRACT

Metrology is advancing by development of new measurement techniques and corresponding hardware. A given measurement technique, however, has fundamental speed and precision limitations. In order to overcome the hardware limitations, we develop signal processing methods based on the prior knowledge that the measurement process dynamics is linear time-invariant.

Our approach is to model the measurement process as a step response of a dynamical system, where the input step level is the quantity of interest. The solution proposed is an algorithm that does real-time processing of the sensor's measurements. It is shown that when the measurement process dynamics is known, the input estimation problem is equivalent to state estimation. Otherwise, the input estimation problem can be solved as a system identification problem. The main underlying assumption is that the measured quantity is constant and the measurement process is a low-order linear time-invariant system. The methods are validated and compared on applications of temperature and weight measurement.


## 1. Introduction

The topic of the paper is a prototypical problem in metrology: a quantity of interest is measured by a measurement device, called a *sensor*. The sensor is a dynamical system with input—the to-be-measured value $u$ (assumed constant during the measurement)—and output—the sensor's reading $y$ (which changes as a function of time):

to-be-measured value $u$ (constant during measurement) $\xrightarrow{\text{sensor}}$ measured value $y$ (exhibits transient response)

Two familiar examples are temperature and weight measurement. They are used to motivate the abstract problem formulation and to test the performance of the methods proposed in the paper.

**Example 1** (*Temperature measurement*). A thermometer is moved to a place with temperature $\bar{u}$. The measured temperature $y$ satisfies Newton's law of cooling

$$\frac{\mathrm{d}}{\mathrm{d}\,t}y = a\left(y - \bar{u}\right), \tag{1}$$

where $a$ is a negative constant that depends on the thermometer and the environment. The differential equation (1) defines a first order linear time-invariant system with input the environmental temperature $\bar{u}$ and output the thermometer's reading $y$. Moving the thermometer from one place to another has the effect of a step input, with initial condition the temperature at the first place. The goal is to measure the temperature at the second place while there is still heat exchange between the thermometer and the environment.

**Example 2** (*Weight measurement*). An object with mass $M$ is placed on a scale with mass $m$. At the time of placing the object, the scale is in a specified (in general nonzero) initial condition. The object placement has the effect of a step input as well as a step change of the total mass of the system—scale and object. The goal is to measure the object's mass while the system is still in vibration.

In the weight measurement application the sensor is the scale. It is modeled as a mass, spring, damper system

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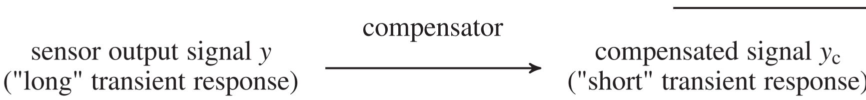
$$(M+m)\frac{\mathrm{d}^2}{\mathrm{d}t^2}y = -ky - d\frac{\mathrm{d}}{\mathrm{d}\,t}y - Mg, \tag{2}$$

where $y$ is the scale's reading, $g$ is the gravitational constant, $k$ is the elasticity constant, and $d$ is the damping constant. In this case, the sensor is a second order linear time-invariant dynamical system with input the measured mass $M$. Note that the system's dynamics depends on the measured mass $M$, which is unknown at the start of the measurement.

The sensor's measurement $y$ exhibits longer or shorter transient, depending on the sensor and the environment. Physically, the transient represents the exchange of mass or energy, which takes place during the measurement process. In metrology, of interest is the steady-state value $\bar{y} = \lim_{t\to\infty} y(t)$, reached in theory only asymptotically. In practice, the measurement is taken after "sufficient decay" of the transient, *i.e.*, when the error $e_y(t) := \| \bar{y} - y(t) \|$ becomes "small". Therefore, there is a *trade-off* between fast measurement (small measurement time $t$) and accurate measurement (small error $e_y$). This trade-off is a *fundamental limitation* of the measurement device determined by the physical laws on the basis of which the sensor is build.

Our goal is to achieve fast and accurate measurement by *predicting* the steady-state value $\bar{y}$ from data $y(t)$ collected over an interval $[0, T]$. The problem is referred to as *dynamic measurement* and the solution is based on digital signal processing. The prediction algorithm, called *compensator*, is a dynamical system with input—the sensor output $y$—and output—the compensated sensor's reading $y_c$:




Ideally, the compensator completely eliminates the transient. In practice, $y_c$ still exhibits a transient. In addition, *disturbances and measurement noise* affect the sensor measurement $y$ even in a steady-state. The scientific challenge of dynamic measurement is to design a compensator that achieves simultaneously short transient response and good disturbance and noise filtering.

### 1.1. State-of-the-art

The *classical approach* in dynamic measurement assumes that the process dynamics is *known* and linear time-invariant, see Eichstädt et al. (2010) for an overview. Consequently, the compensator is also a linear time-invariant system, designed by frequency or time domain de-convolution techniques. The assumption that the process dynamics is known, however, is often unrealistic. In the temperature measurement example, the process dynamics depends on the heat transfer coefficient, which may vary due to unpredictable factors. In the weight measurement example, the process dynamics depends on the unknown mass $M$. In general, the measurement process dynamics depends on the sensor, which is known, and on the environment or the measured quantity, which are unpredictable or unknown.

In order to deal with the issue of the unknown process dynamics in Shu (1993), Niedźwiecki and Wasilewski (1996) an adaptive compensator is proposed. Adaptive methods perform simultaneously online model identification and filtering. The methods of Shu (1993) and Niedźwiecki and Wasilewski (1996) are specifically designed for weight measurement. They need nontrivial modifications for other applications. A model-free method, based on ordinary recursive least-squares estimation, is presented in Markovsky (2015). The computational cost of the model-free method is comparable to the one of a linear time-invariant compensator.

### 1.2. Novelty and contributions

We formulate mathematically the dynamic measurement problem as an input step level estimation problem. Higher order, multivariable measurement processes are considered, which is a generalization over the previously considered in metrology scalar measurement processes. In particular, *sensor fusion* falls into our setting.

In the theoretical development of the methods we use the behavioral approach, where systems are viewed as sets of trajectories rather than equations such as the ones in a transfer function or a state space representation. The ability to switch from one system representation to another is effectively used in the paper to obtain alternative solution methods and to reduce the new problem to solved problems.

In the case of known measurement process (Section 3.1), the input estimation problem is equivalent to a state estimation problem for an augmented autonomous system. The implication of this result is that the Kalman filter, designed for the augmented system is the optimal estimator in the case of Gaussian noise. Deriving the optimal (maximum likelihood) estimator for the dynamic measurement problem with known dynamics is our *first contribution*.

In the case of unknown measurement process (Sections 3.2 and 3.3), the problem is reduced to identification from step response under nonzero initial conditions and identification of an autonomous system with a pole at one. These problems can be solved by existing methods, *e.g.*, the prediction error (Ljung, 1999; Söderström & Stoica, 1989) and the low-rank approximation methods (Markovsky, 2008; Markovsky & Usevich, 2014). Deriving the maximum likelihood estimator for the dynamic measurement problem with unknown dynamics is our *second contribution*.

## 2. Notation and problem statement

### 2.1. Notation

A dynamical system is defined by the set of its trajectories. The statement "$w$ is a trajectory of the system $\mathscr{B}$" is concisely written as "$w \in \mathscr{B}$". We assume that the system under consideration has an input–output partitioning $w = (u, y)$, *i.e.*, the first components of the trajectory are inputs and the remaining ones are outputs.

Let $\sigma$ be the shift operator $(\sigma x)(t) = x(t+1)$. A linear time-invariant system $\mathscr{B}$ admits a state space representation

$$\mathscr{B} = \mathscr{B}_{ss}(A, B, C, D) := \{ w = (u, y) \mid \text{there is } x, \text{ such that } \sigma x = Ax + Bu, \; y = Cx + Du \}. \tag{3}$$

A state space representation $\mathscr{B}_{ss}(A, B, C, D)$ is called minimal if it has the smallest possible dimension of the $A$ matrix. This