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Enlarging parallel robot workspace through Type-2 singularity crossing

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ABSTRACT

In order to increase the reachable workspace of parallel robots, a promising solution consists of the definition of optimal trajectories that ensure the non-degeneracy of the dynamic model in the Type 2 (or parallel) singularity. However, this assumes that the control law can perfectly track the desired trajectory, which is impossible due to modeling errors.

This paper proposes a robust multi-model approach allowing parallel robots to cross Type 2 singularities. The main idea is to shift near singularities to a simplified dynamic model that can never degenerate. The two main contributions are the definition of an optimal trajectory crossing Type 2 singularities and the multi-model control law allowing to track this trajectory. The proposed control law is validated experimentally through a Five-bar planar mechanism.

1. Introduction

Contrary to serial robots, which are largely used in industry, parallel robots are under-represented despite having many advantages, such as higher acceleration capacities and a better payload-to-weight ratio. The small number of parallel mechanisms in factories can be explained by the relative complexity of their model and by the presence of singularities (Arakelian, Briot, & Glazunov, 2008; Conconi & Carricato, 2009; Gosselin & Angeles, 1990), which divide their workspace into different aspects (each aspect corresponding to one or more assembly modes, Merlet, 2006). The manipulator workspace is therefore usually reduced to only one of these aspects, resulting in a greatly reduced reachable workspace size. The main idea of this paper is to propose a control law allowing parallel manipulators to move between those different aspects.

Various types of singularity exist, and for a global overview of the singularity problem the reader is referred to Conconi and Carricato (2009). However, since Type 2 (Gosselin & Angeles, 1990) (or parallel) singularities are probably the most constraining ones, this paper will focus only on this type. In these singularities, one (or more) manipulator's degree of freedom becomes uncontrollable. In order to increase the workspace size several approaches have been envisaged in the literature, such as:

- The design of parallel robots without singularities. This can be done by using the optimal design approach (Briot & Arakelian, 2010; Liu, Wang, & Pritschow, 2006) or by creating decoupled mechanisms (Gogu, 2004; Kong & Gosselin, 2002). This solution is the most usual one, but it usually leads to the design of robots with a small workspace size or robot architectures with very low practicability.
- The use of redundancy (Kurtz & Hayward, 1992; Nahon & Angeles) or, to reduce costs, the use of mechanisms with variable actuation modes (Arakelian et al., 2008; Rakotomanga et al., 2006). These mechanisms can change the way they are actuated without adding additional actuators, but this change can only be carried out when the mechanism is stopped, thus increasing the time necessary to perform the task.
- Planning assembly mode changing trajectories. A first way to do this is to bypass a cusp point (Zein, Wenger, & Chablat, 2008). However, this solution is hardly practical for two main reasons: (i) it forces the mechanism to follow a particular trajectory, which can be very different from the desired one; (ii) only a few mechanisms have cusp points. A second solution is to go directly through a Type 2 singularity (Briot, Arakelian, & Chablat, 2008; Ider, 2005). In Briot et al. (2008), a physical criterion, obtained through the analysis of the dynamic model, is presented. It enables the computation of a trajectory which can cross a singularity without the dynamic model degenerating, by respecting the criterion in question on the singularity locus.

This last solution is promising, since it can considerably increase the workspace size of any parallel mechanism. However, in previous studies it was considered that the controller allowed

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the mechanism to perfectly track the desired trajectory. This is obviously impossible due to modeling uncertainties. In order to fill this gap, the aim of the present paper is to propose an advanced control law dedicated to Type 2 singularity crossing.

Since Type 2 singularities have an impact on the dynamic of the mechanism, the use of a geometric/kinematic controller would not allow taking into account this dynamic degeneracy. Moreover, in Briot et al. (2008) it has been shown that, in order to cross a Type 2 singularity, the mechanism has to track a trajectory that respects a specific criterion on the singularity locus. This criterion gives a relation between the singular position, the mechanism's speed and its acceleration when crossing the singularities. However, only dynamic controllers can perform tracking of velocity and acceleration (Khalil & Dombre, 2004). Most of the different dynamic control loop algorithms can be considered as special cases of the computed torque control (*CTC*) (Craig & Hall, 2005; Khalil & Dombre, 2004; Spong, Hutchinson, & Vidyasagar, 2006). This technique consists of an inner nonlinear compensation loop and an outer loop with an exogenous control signal u. However, this control law is sensitive to modeling errors, so the dynamic model must be well identified (Briot & Gautier, 2012; Gautier, 1997).

When applying a *CTC* control law for singularity crossing, the degeneracy of the dynamic model near the singularity results in computing infinite torques, thus leading to the instability of the controller. No controller has ever been developed for singularity crossing,[1] and most studies concentrate on solutions in order to *avoid* the singularities. In order to be used when crossing a Type 2 singularity, the dynamic model used by the *CTC* must not degenerate near singularities, even if the trajectory does not perfectly respect the physical criterion mentioned above. As a result, in this paper, a new multi-model *CTC* (e.g. see Craig & Hall, 2005; Spong et al., 2006) is proposed, which guarantees that the robot dynamic model of the mechanism does not degenerate near a singularity. This multi-model control law was developed thanks to the definition of a new dynamic criterion based on Briot et al. (2008). The contribution of this paper is to propose a complete methodology, from the trajectory planning to the achievement of singularity crossing on an experimental robot without path restriction.

This paper is organized as follows: first the approach used to compute the criterion for crossing Type 2 singularities is recalled, and a method developed to increase the robustness of the planned trajectory is proposed. Then, in Section 3, the multi-model *CTC* control law developed for crossing singularities is presented. Section 4 introduces the robot used to validate the Type 2 singularity crossing approach proposed. Finally, the relevancy of this controller is demonstrated through full-scale experiments on a Five-bar mechanism.

2. Trajectory generation for crossing a Type 2 singularity

2.1. Dynamic modeling of parallel mechanisms

This section will briefly recall the dynamic equations of a parallel manipulator composed of m links, n degrees of freedom (*dof*) and driven by n actuators. The manipulator is composed of legs attached to the base and to the mobile platform (for a more detailed analysis of closed-loop kinematic chain, the reader is turned to Merlet, 2006). The position and the speed of the manipulator can be fully described using:

- $\mathbf{q} = [q_1, q_2, \ldots, q_n]^T$ and $\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n]^T$ which represent respectively the vectors of active joint variables and active joint velocities,
- $\mathbf{x} = [x, y, z, \phi, \psi, \theta]^T$ and $\mathbf{t} = [\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\psi}, \dot{\theta}]^T$ which are the mobile platform pose parameters and their derivatives with respect to time; x, y and z represent the position of the platform controlled point and ϕ, ψ and θ represent the orientation of the platform about three axes $\mathbf{a}_\phi$, $\mathbf{a}_\psi$ and $\mathbf{a}_\theta$ (*Briant* angles).

Since the mechanism is moving, all of these terms depend on the current time t. However, for purposes of clarity, this dependency will not be written in all equations, and only not time-dependent terms will be specified. Those generalized coordinates are not independent. Indeed, let us consider the vector $\mathbf{v}$ regrouping the independent elements of $\mathbf{t}$. The matrix $\mathbf{D}$ relates the platform twist $\mathbf{t}$ (expressed in the base frame) to the vector $\mathbf{v}$ by (Merlet, 2006)

$$\mathbf{t} = \mathbf{D}\mathbf{v} \tag{1}$$

Note that for mechanism with 6 degrees of freedom, the matrix $\mathbf{D}$ is the identity $[6 \times 6]$ matrix.

Relations between the platform coordinates are found by writing the closed-loop equations. Using Lagrangian formalism, the dynamic model of the mechanism can be written as

$$\tau = \mathbf{w}_b + \mathbf{B}^T \lambda, \tag{2}$$

$$\mathbf{w}_p = \mathbf{A}^T \lambda \tag{3}$$

where

- τ is the $[n \times 1]$ vector of the input efforts,
- λ is the $[n \times 1]$ vector of the Lagrange multipliers,
- $\mathbf{A}$ and $\mathbf{B}$ are two $[n \times n]$ matrices deduced from the mechanism loop-closure equations, such that $\mathbf{A}\mathbf{v} = \mathbf{B}\dot{\mathbf{q}}$ (Merlet, 2006),
- $\mathbf{w}_b$ and $\mathbf{w}_p$ are $[n \times 1]$ terms related to the Lagrangian L of the system by

$$\mathbf{w}_b = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial L}{\partial \mathbf{q}}, \quad \mathbf{w}_p = \frac{d}{dt}\left(\frac{\partial L}{\partial \mathbf{v}}\right) - \frac{\partial L}{\partial \mathbf{x}} \tag{4}$$

In this expression, $\mathbf{w}_p$ is the wrench applied to the platform by the legs and the external forces (Briot et al., 2008) and t is the time.

Then, assuming that matrix $\mathbf{A}$ can be inverted and by substituting (3) into (2), the general dynamic model of parallel manipulators is obtained (Khalil & Dombre, 2004):

$$\tau = \mathbf{w}_b + \mathbf{J}^{T\,0}\mathbf{w}_p, \tag{5}$$

where

- ${}^0\mathbf{w}_p$ is the expression of the wrench $\mathbf{w}_p$ in the base frame, i.e. ${}^0\mathbf{w}_p = \mathbf{D}\mathbf{w}_p$,
- $\mathbf{J} = {}^0\mathbf{A}^{-1}\mathbf{B}$ is the matrix relating the platform twist $\mathbf{t}$ and $\dot{\mathbf{q}}$, with ${}^0\mathbf{A}$ the expression of matrix $\mathbf{A}$ in the base frame, i.e. ${}^0\mathbf{A} = \mathbf{A}\mathbf{D}^{-1}$.

2.2. Type 2 singularity crossing

Based on the analysis of the kinematic model, a classification of singularities into three different types is proposed in Gosselin and Angeles (1990):

- *Type* 1 *singularities or serial singularities* occur when the mechanism is in a position such that the kinematic matrix $\mathbf{B}$ becomes rank deficient. In such configurations, the mechanism loses its ability to move in one given direction.

[1] Note that a possible solution for crossing singularities is to plan a fast trajectory toward the singularity locus. Once the mechanism is close enough from the singularity, the controller could declutch the actuators, and couple them back once the mechanism is far enough from the singularity. This solution is obviously not robust at all and presents many disadvantages.