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Estimating the frequency response of a system in the presence of an integrator

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ABSTRACT

A system with an integrator is one which does not have a steady-state gain at zero frequency. An example is a battery; when subjected to a constant charge or discharge current the voltage will continuously increase or decrease until the cell reaches its maximum/minimum cut-off voltage and not reach a steady-state value. Frequency response estimation techniques that minimises leakage errors lead to significant errors at low frequencies of the response. This paper develops and presents a technique whereby the low frequency errors are eliminated. The technique is applied over the frequencies of interest, except DC frequency, and gives better results over windowing and a local polynomial frequency response estimation method. As such, an accurate low frequency response and noise power spectrum can now be obtained which in turn can be used for estimating accurate parametric models.

1. Introduction

The topic of system identification deals primarily with the derivation of dynamical models of systems from measured time signals or data. Obtaining a parametric dynamic model from the data generally requires assumptions regarding the model structure and order. Alternatively, the frequency response function (FRF) is a non-parametric estimate of a linear system's dynamics. The FRF can then be used to estimate a parametric model and because it reveals the main features of the dynamics it will also assist in the parametric model order and structure selection (Schoukens, Vandersteen, Barbe, & Pintelon, 2009).

If the input signal is periodic and the system under test is linear and stable, an accurate estimate of the frequency response can be obtained at all frequencies. If the input is aperiodic, an error known as the leakage error is introduced when relating the input and output spectra while using finite time records. The classical approach to reduce leakage is to use a windowing function (e.g. Hanning) with the input and output time records (Blackman & Tukey, 1958) and furthermore the number of blocks for averaging can be increased by introducing block overlapping (Barbe, Pintelon, & Schoukens, 2010; Widanage, Douce, & Godfrey, 2009).

In more recent work, techniques such as the Local Polynomial Method (LPM) (Pintelon, Schoukens, Vandersteen, & Barbe, 2010), the Local Rational Method (LRM) (McKelvey & Guérin, 2012) and TRansient Impulse Response Modelling Method (TRIMM) (Hägg & Hjalmarsson, 2012; Hagg, Hjalmarsson, & Wahlberg, 2011) have been developed. These make use of the mathematical structure of the leakage error, for example LPM and LRM make use of the leakage error smoothness and reduce the error by approximating it over a narrow frequency window with a polynomial in the case of LPM or rational function in LRM.

Systems that consist of an integrator have a well defined frequency response at every frequency, except at DC frequency (0 rad/s) at which the FRF tends to infinity. In its transfer function this corresponds to having a pole at zero. If an input is applied that has a non-zero mean value the DC frequency is perturbed and the output will have a trend that increases or “drift”. In theory, due to the infinite gain at DC frequency, the Fourier transform of a system with an integrator does not exist.

In practice, however, an experiment is performed and data are collected over a *finite* time period. The drift term in the measured output signal, which more specifically is a ramp signal of finite length, has a well defined Short Term Fourier Transform (STFT). The spectrum of this ramp signal if ignored introduces an error that is not due to leakage and the use of methods such as windowing or the polynomial approximation method, which are suited for reducing leakage effects, are unable to minimise the drift error, giving biased estimates, particularly at the low frequencies.

A lithium ion battery is an example of a system with an integrating effect. As an energy storage element its terminal voltage gradually increases or decreases to the applied charge/discharge current. Impedance estimates of a cell are a useful

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non-invasive measure for cell ageing, fault and state-of-charge diagnostics (Offer, Yufit, Howey, Wu, & Brandon, 2012; Tröltzsch, Kanoun, & Tränkler, 2006). The current and voltage are measured whilst the cell is in open-loop, often in a laboratory or in a vehicle by a Battery Management System (BMS) (Howey, Yufit, Mitcheson, Offer, & Brandon, 2013). The current usually has no DC offset to avoid perturbing the 0 frequency, if however, the 0 frequency is excited the current and voltage are measured over a very short time interval whereby any integrating effect is ignored and the impedance or any estimated impedances below a certain frequency (<1 Hz) are discarded (Howey et al., 2013).

The results of Howey et al. (2013) show good promise for on-vehicle cell impedance estimation; which used windowing for leakage error suppression and calculating the cross and auto-power spectra for impedance estimation and the low frequency estimates were discarded by evaluating a coherence function. In contrast to their work, the solution strategy developed and described in this paper uses LPM for leakage suppression and further modifications to the output spectrum which together can be used to improve the low frequency impedance estimation, without discarding them, when a cell is under a DC bias and by treating it as a system with an integrator.

The framework within which the estimation technique is developed and presented here assumes a continuous time system in open-loop with measurement noise in the output (an output error problem). The structure of the paper is organised as follows. The following section, with the aid of a simulation example, highlights the low frequency errors observed when estimating the frequency response. Sections 3 and 4 describe how the low frequency error can be significantly reduced and the power spectrum of the disturbing noise be estimated. Section 5 shows how the developed method improves the frequency response when applied to a battery and Sections 6 and 7 analyses several aspects of the developed technique followed by conclusions of the method.

2. System and problem setting

A general continuous time system with an integrator in an output-error framework (only the system output is corrupted with noise) is considered in this paper and the system set-up within which the solution is developed is shown in Fig. 1. The input (for example a current signal) is assumed to be a computer generated sequence of samples that passes through a zero-order hold (ZoH) and is assumed to be noise free. The input can be an arbitrary deterministic or random signal but persistently exciting over the frequency of interest. The output from the system (for example a voltage signal) is corrupted with a filtered white noise process and

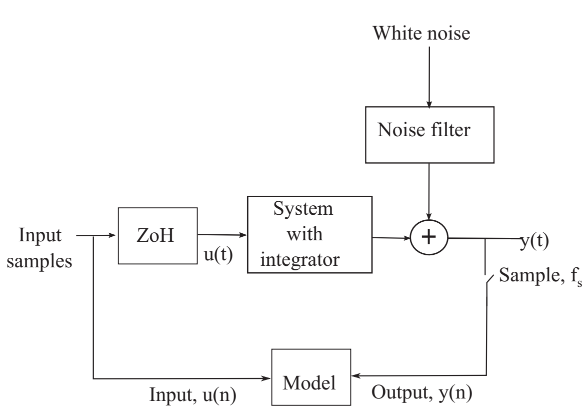

Fig. 1. System set-up. Both input and output data are available for identification and only the output is assumed to be corrupted with coloured noise.

is zero mean and stationary (see the following equation):

$$y(t) = g(t) * u(t) + h(t) * e(t) \tag{1}$$

In Eq. (1) "$*$" denotes the convolution operator, $u(t)$ is the continuous input signal (after passing through the ZoH) and $e(t)$ is a zero mean Gaussian white noise process with variance σ_e^2, uncorrelated with the input. Further, $g(t)$ and $h(t)$ are respectively the system and noise filter impulse responses.

The system is perturbed and measured over a finite time length of $0 \le t \le L$ seconds and the objective is to estimate the frequency response of the continuous time system and the power spectrum of the disturbing noise from the known input and output time sampled data (sampled at a rate of f_s Hz). An accurate estimate of the noise variance at each frequency is important, as it can be used as a frequency weighting in a subsequent parametric estimate of the system transfer function (Schoukens, Rolain, Vandersteen, & Pintelon, 2011).

A simulation example is presented in the following subsection to illustrate the level of error encountered when applying two existing frequency response estimation methods, windowing and LPM.

2.1. A simulation example

Consider the following transfer function ($G(s)$), which is a linear system consisting of an integrator:

$$G(s) = \frac{4s + 0.05}{(5s+1)s} \tag{2}$$

The system is excited with a normally distributed random signal with unit mean and a standard deviation of 2, $u(n) \sim \mathcal{N}(1, 2^2)$. The noise filter ($H(s)$) is a low pass Chebyshev Type I filter of order 10 and with an angular pass band frequency of 0.2π rad/s. A zero mean normally distributed signal with a standard deviation of 0.5 is filtered through $H(s)$ and added to the system output as coloured noise. With a sampling frequency of 1 Hz, $N = 1000$ input and output samples are acquired for the frequency response estimation.

2.2. Classical estimation via windowing

The classical method of estimating a frequency response is to segment the time data record into equal blocks by multiplying it with a window, such as a Hanning window which also minimises leakage errors, and compute the ratio of the cross-power spectral density and the auto-power spectral density (Bendat & Piersol, 1993; Blackman & Tukey, 1958; Schoukens, Rolain, & Pintelon, 2005). In order to have a higher frequency resolution and access to low frequency content, the data record on this occasion, is not segmented. Instead a Hanning window is applied over the N sampled values and an estimate of the frequency response is obtained based on one block. Denoting $u(n)$ as the input samples, the windowed input is $u_w(n) = u(n) \times w(n)$, where $w(n) = 1 - \cos((2\pi n f_s)/N)$ is the Hanning window function. Similarly, $y_w(n)$ is the windowed sampled output signal.

When using windowing methods the input and output Discrete Fourier transform (DFT) can in general be related as

$$Y_w(k) \approx G(\omega_k) U_w(k) + V(k) \tag{3}$$

In Eq. (3) $Y_w(k)$ is the DFT of $y_w(n)$ at the kth harmonic number, $G(\omega_k)$ is the frequency response evaluated at the discrete angular frequency $\omega_k \triangleq 2\pi k f_s/N$ and $V(k)$ is the noise term uncorrelated with the input. The equation is an approximation, since windowing reduces but does not eliminate leakage errors. Due to the ZoH effect, $U_w(k)$ is the product of the DFT of the windowed input ($u_w(n)$) and the spectrum of a ZoH.[1] From Eq. (3) an estimate for the frequency response is $\hat{G}(\omega_k) = Y_w(k)/U_w(k)$. The magnitude in dB and the phase of the estimated response along with the true

[1] ZoH spectrum at kth harmonic, $Z(k) = |\sin(\pi k/N)/(\pi k/N)| e^{-j\pi k/N}$.