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Non-diagonal $\mathcal{H}_\infty$ weighting function design: Exploiting spatio-temporal deformations in precision motion control

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ABSTRACT

Model-based control design requires a careful specification of performance and robustness requirements. In typical norm-based control designs, performance and robustness requirements are specified in a scalar optimization criterion, even for complex multivariable systems. This paper aims to develop a novel approach for the formulation of this optimization criterion for multivariable motion systems that exhibit spatio-temporal deformations. To achieve this, characteristics of the underlying system are exploited to design multivariable weighting functions. In contrast to pre-existing approaches, which typically lead to diagonal weighting functions, the proposed approach enables the design of non-diagonal weighting functions. Extensive experimental results confirm that the proposed procedure can significantly improve the performance of an industrial motion system compared to earlier approaches.

1. Introduction

The design of a high-performance controller for a complex multivariable system hinges on the specification of a suitable optimization criterion. In model-based control, a scalar criterion is typically adopted that should reflect the user-defined performance requirements, as in Zhou, Doyle, and Glover (1996), McFarlane and Glover (1990) and Skogestad and Postlethwaite (2005). These requirements are defined in the optimization criterion by means of weighting functions, see, e.g., Lanzon and Tsiotras (2005), Hu, Bohn, and Wu (2000), Graham and de Callafon (2007) and Lundström, Skogestad, and Wang (1991). Relevant examples of model-based control include $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control.

The key advantage of $\mathcal{H}_\infty-$optimization is that it is capable of delivering robust controllers by explicitly taking model uncertainty into account. In contrast, LQG and $\mathcal{H}_2$ designs have no guaranteed robustness margins, as is shown in Doyle (1978). Besides the ability of $\mathcal{H}_\infty-$optimization to design robust controllers, it allows for the design of weighting functions using loop-shaping concepts, see, e.g., Doyle and Stein (1981) and McFarlane and Glover (1990). These techniques are particularly suitable for motion control, where controllers are traditionally being designed using manual loop-shaping, see, e.g., van de Wal, van Baars, Sperling, and Bosgra (2002).

Weighting function design for motion systems typically employs diagonal weighting functions to specify performance requirements, see, e.g., Steinbuch and Norg (1998), Schönhoff and Nordman (2002) and van de Wal et al. (2002). The underlying assumption for this approach is that the system is approximately diagonal. This assumption is in general not valid for multivariable motion systems that exhibit spatio-temporal deformations. For such systems, parasitic dynamics are typically relevant in more than one channel of the servo system. In fact, such dynamics have in general a specific directional effect, making the control problem inherently multivariable. In such cases, non-diagonal weighting selection in the performance channels of the $\mathcal{H}_\infty-$optimization problem might be very effective to enhance the performance of multivariable motion systems.

Although $\mathcal{H}_\infty$ control is promising for the design of controllers for multivariable motion systems that exhibit spatio-temporal dynamics, at present there is no procedure to adequately specify the control goal in standard $\mathcal{H}_\infty$ control criteria. This paper aims to improve performance of such systems by exploiting non-diagonal weighting functions that address the directional effect of spatio-temporal dynamics. To achieve this, the designed weighting functions incorporate frequency-localized compensation of directionality in the system. The performance improvement obtained with non-diagonal weighting functions is illustrated by means of an experimental case study for an industrial high-performance motion system. This paper is an extension of a previously published conference paper (Boeren, van Herpen, Oomen, van de Wal,

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& Bosgra, 2013) and includes a complete derivation, analysis and experimental results.

This paper is organized as follows. In Section 2, loop-shaping for multivariable systems is revisited. In Section 3, a procedure is proposed to determine transformation matrices that provide frequency-localized compensation of the directionality in the system. In Section 4, multivariable weighting functions are proposed to specify performance requirements for motion systems. In Section 5, extensive experimental results for an industrial motion system are provided to evaluate the achievable performance enhancement of the proposed weighting function design. Finally, conclusions are provided in Section 6.

Notation: Let $P(s) \in \mathcal{R}^{n \times n}$ denote a square real-rational transfer function matrix, with s the Laplace operator. For ease of notation, $P(s)$ is also denoted P. Furthermore, let the singular values of a matrix $A \in \mathbb{C}^{n \times m}$ be denoted by $\sigma_i(A)$, with the maximum (resp. minimum) singular value denoted by $\overline{\sigma}(A)$ (resp. $\underline{\sigma}(A)$). The eigenvalues of a matrix $A \in \mathbb{C}^{n \times m}$ are denoted by $\lambda_i(A)$. A matrix $U \in \mathbb{C}^{n \times n}$ is unitary if $U^H U = UU^H = I$, where U^H is the conjugate transpose of U.

Remark: In order to clearly illustrate the concepts in this paper, attention is restricted to square systems $P(s)$. Extensions to non-square systems are conceptually straightforward.

2. Problem definition

2.1. Loop-shaping

Closed-loop performance and robustness requirement can often be translated in a desired loop transfer function. The goal of loop-shaping is to attain this desired loop transfer function $L_{des} = PC$, with P being the system and C being the controller, that prescribes the desired gains of the system as a function of frequency. In particular, three requirements for L_{des} are commonly imposed, see, e.g., McFarlane and Glover (1990, Chap. 6) and Skogestad and Postlethwaite (2005, Chap. 9), These requirements are indicated in Fig. 1 by the black triangles and the cross-over region $f \in [f_1, f_2]$, given by $\underline{\sigma}(L_{des}(f_1)) = 1$ and $\overline{\sigma}(L_{des}(f_2)) = 1$.

R1. Nominal stability: The maximum roll-off rate of $|\lambda_i(L_{des})|$, $\forall i$ in the cross-over region $f \in [f_1, f_2]$ is -40 db/decade.
R2. A large open-loop gain needs to be attained for frequencies below f_1, i.e., $\underline{\sigma}(L_{des}) \gg 1 \;\; \forall f \in [0, f_1]$.
R3. A small open-loop gain needs to be attained for frequencies above f_3, i.e., $\overline{\sigma}(L_{des}) \ll 1 \;\; \forall f \in [f_3, f_\infty]$.

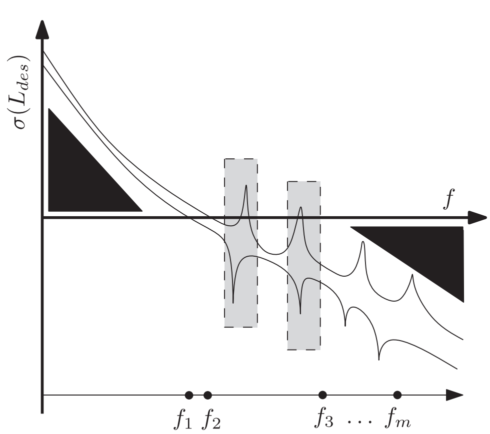

Fig. 1. Singular values of the desired loop transfer function $L_{des}(s)$ for performance and robustness.

Requirement R1 ensures nominal closed-loop stability, while R2–R3 reflect classical closed-loop performance and robustness requirements. To illustrate the connection between R2–R3 and closed-loop requirements, consider the desired sensitivity function $S_{des} = (I + L_{des})^{-1}$ and desired complementary sensitivity function $T_{des} = L_{des}(I + L_{des})^{-1}$. As shown in McFarlane and Glover (1990), these expressions can be approximated in the relevant frequency ranges by

$$\overline{\sigma}(S_{des}) \le \frac{1}{\underline{\sigma}(L_{des})} \ll 1 \quad \text{where } \underline{\sigma}(L_{des}) \gg 1,$$
$$\overline{\sigma}(T_{des}) \le \overline{\sigma}(L_{des}) \ll 1 \quad \text{where } \overline{\sigma}(L_{des}) \ll 1. \tag{1}$$

Expression (1) reveals that L_{des} implicitly determines the singular values of S_{des} and T_{des}. Complying with typical requirements, low-frequency disturbances are attenuated if $\overline{\sigma}(S_{des}) \ll 1$, while high-frequency robustness with respect to model uncertainty is obtained if $\overline{\sigma}(T_{des}) \ll 1$. This result shows that R2–R3 dictate closed-loop performance and robustness requirements.

In this paper, the $\mathcal{H}_\infty$ loop-shaping design procedure presented in McFarlane and Glover (1990) is used to attain L_{des}.

Goal 1. *Given $\sigma_i(L_{des})$, the goal in loop-shaping is to design weighting functions $W_1(s), W_2(s)$ such that*

$$\sigma_i(P_s) \approx \sigma_i(L_{des}) \quad i = 1, \dots, n.$$

where the shaped system P_s is given by

$$P_s(s) = W_2(s) P(s) W_1(s). \tag{2}$$

By loop-shaping $\sigma_i(P_s)$, Goal 1 ignores nominal closed-loop stability considerations as given in R1. In the $\mathcal{H}_\infty$ loop-shaping design procedure, closed-loop stability is ensured by subsequently using $\mathcal{H}_\infty$–optimization based on the designed $P_s(s)$. A complete tutorial for the design of controllers using the $\mathcal{H}_\infty$ loop-shaping design procedure is provided in McFarlane and Glover (1990).

Remark 1. The open-loop weighting functions $W_1(s), W_2(s)$ in (2) can be directly translated into equivalent closed-loop weighting functions, as is used in common $\mathcal{H}_\infty$–optimization algorithms including Skogestad and Postlethwaite (2005).

2.2. Directionality in multivariable systems

For multivariable systems, the input and output directionality of $P(s)$ complicates the design of $W_1(s)$ and $W_2(s)$. This directionality determines the connection between the singular values and the individual entries of $P(s)$, as reflected in the singular value decomposition. The singular value decomposition at frequency ω_k is given by

$$P(j\omega_k) = U(j\omega_k) \Sigma(j\omega_k) V^H(j\omega_k), \tag{3}$$

with $\Sigma(j\omega_k) = \text{diag}(\sigma_1(j\omega_k), \sigma_2(j\omega_k), \dots, \sigma_n(j\omega_k)) \in \mathbb{R}^{n \times n}$, and unitary matrices $V(j\omega_k) \in \mathbb{C}^{n \times n}$ and $U(j\omega_k) \in \mathbb{C}^{n \times n}$.

In (3), Σ represents the singular values of the system, while V and U represent the corresponding input and output directionality. The key point is that this directionality is frequency dependent. Since performance and robustness requirements are specified by means of loop-shaping Σ, the directionality of P as defined in V and U should be accounted for in $W_1(s)$ and $W_2(s)$, as illustrated in Fig. 2. As a result, the singular values in Σ are accessible for loop-shaping.

In classical control design, directionality is often only addressed in the cross-over region, see, e.g., Maciejowski (1989, Section 4.3). Typically, this design methodology is focused on determining static transformation matrices $T_u, T_y \in \mathbb{R}^{n \times n}$ such that

$$P_{diag}(s) = T_y P(s) T_u, \tag{4}$$