Control Engineering Practice 18 (2010) 1003–1012

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Linear, parameter-varying control of a supercavitating vehicle

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ARTICLE INFO

Article history:
Received 23 March 2009
Accepted 25 April 2010
Available online 26 May 2010

Keywords:
Supercavitation
LPV control
6-DOF control
Underwater vehicles
Nonlinear dynamics

ABSTRACT

A systematic approach to parameter-dependent control synthesis of a high-speed supercavitation vehicle (HSSV) is presented. The aim of the control design is to provide robust reference tracking across a large flight envelope, while directly accounting for the interaction of liquid and gas phases with the vehicle. A nonlinear dynamic HSSV model is presented and discussed relative to the actual vehicle. A linear, parameter-varying (LPV) controller is synthesized for angle rate tracking in the presence of model uncertainty. The control design takes advantage of coupling in the governing equations to achieve improved performance. Multiple LPV controllers synthesized for smaller overlapping regions of the parameter space are blended together, providing a single controller for the full flight envelope. Time-domain simulations implemented on high-fidelity simulations, provide insight into the performance and robustness of the proposed scheme.

1. Introduction

The velocity of conventional underwater vehicles is limited by the drag induced by skin friction, the interaction of liquid with the vehicle surface. Since drag increases exponentially with velocity, the amount of thrust propelling an underwater vehicle has to increase exponentially to achieve increase in speed. Due to limits on propulsion, current underwater vehicles are limited to approximately 50 m/s. Russian designers in the 1970s proposed a radically different approach to reduce drag on underwater vehicles (Logvinovich, 1972). The vehicle surface in contact with the fluid was reduced by enveloping the vehicle in a gas cavity. The water vapor cavity generated by *supercavitation* led to the Skhval underwater vehicle (Ashley, 2001) which can reach speeds up to 100 m/s. The U.S. Navy is pursuing a major supercavitating vehicle development program (Fig. 1). The Underwater Express program funded by DARPA aims to develop a supercavitating submarine. Supercavitation, to reduce friction on underwater objects, is not widespread since navigation and control are fundamental challenges for these vehicles. The nonlinear equations of motion of supercavitating vehicles are highly coupled and function of the cavity evolution. Even the simplistic single degree-of-freedom (1-DOF) model, Kirschner, Rosenthal, and Uhlman (2003), points out the importance of the cavity delay dependence. The problem of controlling the full 6-DOF nonlinear equations of motion for the a supercavitating vehicle is described by Kirschner, Kring, Stokes, and Uhlman (2002), Goel (2002), and Kurdila, Lind, Dzielski, Jammulamadaka, and Goel (2003). Linear quadratic regulator (LQR) control techniques are used to stabilize the 6-DOF vehicle dynamics. The results are valid only around a small vicinity of the straight flight operating point. Several results were published pointing out the importance of applying nonlinear control design techniques for supercavitating vehicles (Dzielski & Kurdila, 2003; Lin, Balachandran, & Abed, 2006; Vanek, Bokor, Balas, & Arndt, 2007). These results are limited to longitudinal motion and omit important dynamical properties of the vehicle, including the cross coupling of the asymmetric fin immersion with the gas cavity.

The present article provides an approach to control of the 6-DOF nonlinear supercavitating vehicle using linear, parameter-varying (LPV) control techniques. This method developed by Wu (1995), Wu, Yang, Packard, and Becker (1996), and Apkarian, Gahinet, and Becker (1995) has the benefit of being able to account for a large flight envelope, and draws on the knowledge and experience of the robust control field. The analytical and numerical complexity of the LPV controller synthesis for the 6-DOF coupled dynamics is handled using systematic design tools unlike in most nonlinear design techniques. In Section 2 the mathematical model of the vehicle is briefly described, more details can be found in Vanek (2008). The LPV control design technique is summarized in Section 4. Special attention is given to exploring the capabilities of the LPV controllers which are implemented in a high-fidelity mathematical model of the vehicle in Section 6. Conclusions and recommendations are given in Section 7.

2. Mathematical model of the HSSV

The dynamical behavior of a supercavitating vehicle is complex due to the gas cavity surrounding the hull. This

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doi:10.1016/j.conengprac.2010.04.006

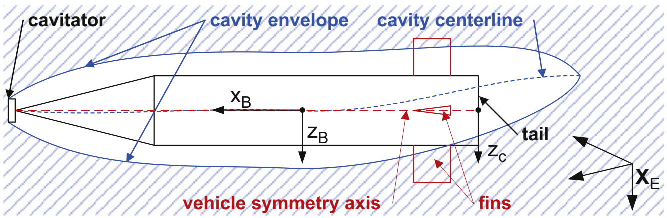

Fig. 1. Vehicle configuration (side view) of the ONR test bed.

represents a significant challenge for the control designer since the vehicle, including the control surfaces, has to operate under the impact of two fundamentally different media, liquid and gas. The highly nonlinear vehicle dynamics have led researchers to make simplifying assumptions regarding the vehicle dynamics models (Kirschner et al., 2003).

The approach taken in this article is to represent the nonlinear 6-DOF vehicle dynamics with linear, parameter-varying (LPV) model. The LPV technique offers a systematic design methodology, where performance, noise and uncertainties can be treated the same way as in the $\mathcal{H}_\infty$ framework, to address control of highly coupled, nonlinear uncertain dynamical systems. Experience with linear robust control can be directly translated to the LPV design process, unlike in many nonlinear approaches where precise knowledge of the plant is inevitable, while the desired response is hard to tune. Many finite dimensional systems can be well characterized with LPV systems, where the dominant underlying dynamics are understood while the state-space description involves *exogenous* variables, with the following properties: the dynamic evolutionary rules for the exogenous variables behavior is not understood, or is too complicated to be modeled; and the values of the exogenous variables effect, in a known manner, the evolution rules governing the dynamics of the state variables; the values of the exogenous variables change with time. LPV control techniques have been applied successfully to a number of advanced, high performance aircraft, missiles, flexible structures and road vehicles (Apkarian et al., 1995; Balas, 2002; Ganguli, Marcos, & Balas, 2002; Poussot-Vassal et al., 2008) since real-time implementation of these controllers is similar to that of existing gain-scheduled controllers.

The vehicle configuration used in this article is similar to existing underwater vehicles with two set of control surfaces, a cavitator at the front and fins in the aft (Fig. 1). If the cavitator, responsible for generating the gas cavity, has a single degree of freedom in pitch, the vehicle must use bank-to-turn maneuvers for trajectory tracking. A two-degree of freedom cavitator allows skid-to-turn maneuvers. The later configuration is more advantageous from a control design perspective since, disturbance attenuation in yaw channel is difficult with only fin control. The vehicle dynamics exhibits non-minimum phase response in the lateral plane when only fin control inputs are available. This severely restricts the achievable control bandwidth and disturbance attenuation properties of the control design (Freudenberg & Looze, 1985).

The vehicle motion has six degrees of freedom, and 12 states are required to describe the equations of motion of the vehicle in inertial frame

$$\mathbf{f}_B = m(\dot{\mathbf{v}}_B + \omega \times \mathbf{v}_B) \quad (1)$$

$$\mathbf{m}_B = \mathbf{I}\dot{\omega} + \omega \times (\mathbf{I}\omega) \quad (2)$$

$$\dot{\Phi} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \dfrac{\sin(\phi)}{\cos(\theta)} & \dfrac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \omega \quad (3)$$

$$\dot{\mathbf{x}} = DCM_{B\to E}\mathbf{v}_B \quad (4)$$

where m is the mass of the vehicle, $\mathbf{I}$ is the inertia matrix, $\mathbf{v}_B = [u,v,w]^T$ and $\omega = [p,q,r]^T$ denote the linear velocity of the center of gravity (c.g.) and angular velocity of the body, respectively. $\mathbf{f}_B$ and $\mathbf{m}_B$ are the resultant of applied forces and moments acting on the body. The position $\mathbf{x}$ and orientation Φ of the body with respect to the inertial frame are the standard kinematic relations. The vehicle states are: angle-of-attack (α [rad] $= \arctan(w/u)$, the angle between surge and heave velocity), sideslip angle (β [rad] $= \arctan(v/u)$, the angle between surge and sway velocity), inertial velocity (V_i [m/s] $= \sqrt{u^2+v^2+w^2}$), roll- (p [rad/s]), pitch- (q [rad/s]), yaw-rate (r [rad/s]) all in body frame, North- (x [m]), East- (y [m]), Down-position (z [m]), bank- (ϕ [rad]), attitude- (θ [rad]), heading-angle (ψ [rad]) in Earth frame. The sum of forces $\mathbf{f}_B$ [N] and moments $\mathbf{m}_B$ [N/m] acting on the vehicle can be written as

$$\mathbf{f}_B = \mathbf{f}_t + \mathbf{f}_c + \sum_{i=1}^{4} \mathbf{f}_f^i + \mathbf{f}_p + \mathbf{f}_g \quad (5)$$

$$\mathbf{m}_B = \mathbf{f}_c \times \begin{bmatrix} L_c \\ 0 \\ 0 \end{bmatrix} + \sum_{i=1}^{4} (\mathbf{f}_f^i \times L_f^i + m_f^i) + \mathbf{f}_p \times L_p + \mathbf{m}_p \quad (6)$$

where the thrust force $\mathbf{f}_t$ assumed to act along the body x axis, hydrodynamic forces on the cavitator ($\mathbf{f}_c$) are function of vehicle states and cavitator pitch and yaw deflection angles ($\delta_{c,p}, \delta_{c,y}$). The forces on the four fins ($\mathbf{f}_f^i$) are slope-discontinuous functions, obtained from a lookup table, of vehicle states, fin deflection angles (δ_f^i), sweepback angles and relative immersions of the fins. The gravitational force on the c.g. ($\mathbf{f}_g$) always points towards the Earth z axis. Contact of the vehicle with the fluid is called *planing*. Planing results in a large impulse force ($\mathbf{f}_p$) to direct the body back into the cavity. It can lead to oscillating motion like a fast boat bouncing on the top of water. There are two distinct modes of the vehicle dynamics: (i) the entire vehicle inside the cavity, no forces are generated by planing, (ii) the transom immersed into the liquid outside of the cavity. In the later case the resulting planing force acts in the opposite direction of the immersion, hence it does not generate moment around the body x axis like the cavitator and thrust forces. The moment arms of the fin forces L_f^i and the center of pressure of the planing L_p are also functions of vehicle states and relative cavity position. The equations of motion can be propagated by integration of the rigid body dynamics of (1)–(4). For further details on the nonlinear equations of motion the reader is referred to Vanek (2008).

2.1. Logvinovich cavity model

The presence of the cavity bubble is the main difference between an airborne missile and a supercavitating vehicle. The cavity model used in the article uses the independence principle assumption of Logvinovich (1972): "Each cross-section of the cavity expands relative to the path of the body-center almost independently of the subsequent or preceding motion of the body." As a result, distortions of the cavity caused by motion of the cavitator, which marks the centerline of the cavity bubble, propagate towards the afterbody with a time lag, also known as memory-effect (Kurdila et al., 2003), proportional to the vehicle length (L) and inversely proportional with speed (V_i). Hence, the dynamic behavior of the vehicle is influenced not only by instantaneous states but also past vehicle states, particularly by the past trajectory of the cavitator. Since the cavity is coupled with the vehicle motion through memory effect, the understanding of cavity–vehicle interaction is of great importance to vehicle stability and control. The cavity shape at the transom region determines immersion of the fins and planing.