Control Engineering Practice 18 (2010) 1379–1385

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Analysis of synchronized coupled oscillators with application to radar beam scanning

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ARTICLE INFO

Article history:
Received 26 January 2010
Accepted 23 July 2010
Available online 13 August 2010

Keywords:
Coupled oscillator array
Radar beam scanning
Nonlinear dynamics
Stability
Randomness
Monte Carlo simulation

ABSTRACT

The stability and nonlinear behavior of synchronized coupled oscillators are studied via nonlinear control theory and applied to radar beam scanning arrays. The analysis indicates that only one stable equilibrium point exists when choosing a specific set of free running frequencies, and it is associated with the desired phase shift, but within a given range of values. Simulation results show that radar beam scanning arrays of oscillators with strong coupling have better angular resolution than arrays with weak coupling, and these arrays are more robust under the influence of randomness of the free running frequency.

1. Introduction

For traditional radar beam scanning array, a phase shifter is used with each antenna element to establish a constant phase progression along the antenna array. A constant phase progression will force the electromagnetic wave to add up so that the energy radiates at a particular angle to the array. Since the 1980s, Monolithic Microwave Integrated Circuits (MMICs) have attracted much attention due to their reproducibility and smaller dimension. However, it is difficult to integrate the bulky phase-shifters in the monolithic module along with other microwave circuitries, such as amplifiers, power distribution network, and DC bias network, especially when the application involves a large beam scanning array.

Recently, a new radar beam scanning technique using array of coupled oscillators was demonstrated (Georgiadis, Collado, & Suárez, 2006; Hwang & Myung, 1998; Liao & York, 1993; Pogorzelski, Maccarini, & York, 1999; Shen & Pearson, 2004). This alternative approach to the applications of radar beam scanning arrays is to use coupled oscillators for achieving the constant phase progression along the array and thus avoids any use of phase shifters. This technique can reduce the complexity of phase control circuits and ease the integration of phased array. Consequently, it simplifies the architecture of the T/R module and reduces the overall cost. The concept of this new technique is that the array phase distribution, and hence the radar beam scanning angle, can be controlled by detuning free running frequencies of oscillators. By applying this mathematical model, it was demonstrated that a desired beam angle can be achieved by simply detuning the free-running frequencies of two oscillators on the edges for four-element (Liao & York, 1993) and six-element one-dimensional arrays (Liao & York, 1994a,c) with uniform amplitude distribution.

The nonlinear dynamics of one-dimensional coupled oscillator array can be analyzed via two different approaches, either in the time domain (York, 1993) or the frequency domain (Georgiadis et al., 2006; York, Liao, & Lynch, 1994). In Georgiadis et al. (2006), a semi-analytical approach based on the harmonic balance (HB) was presented using the auxiliary-generator technique (Collado, Ramírez, Suárez, & Pascual, 2004; Suárez & Quéré, 2003) and was compared with the Full HB analysis and envelope-transient method (Ngoya & Larcheveque, 1996). The closed-loop transfer function was presented and the stability was discussed by examining the eigenvalues of the transfer function. Since multiple solutions may exist, it cannot be guaranteed that this nonlinear system always has only one single stable solution and the system stability was also discussed in Liao and York (1993), Nogi, Lin, and Itoh (1993), and York (1993). In practice, due to the fabrication tolerance of the oscillator, the free running frequency can randomly deviate from the desired value. Such randomness of the free running frequencies can cause errors of the phase shifts between the adjacent elements, and hence cause an error of the main beam scanning angle (EMBSA) in the array (Shen & Pearson, 2005).

In this paper, by employing the time domain method, a new approach to the analysis of system behaviors of one-dimensional

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doi:10.1016/j.conengprac.2010.07.001

coupled oscillator arrays based on nonlinear control theory is presented. Stability of the nonlinear coupled oscillator array system is investigated by examining the equilibrium points analytically and numerically. Important nonlinear phenomena are also demonstrated by 2-D and 3-D phase portraits for the practical design of radar beam scanning array using coupled oscillators that were built and tested in Liao and York (1993). Note that previous work stops at the formulation of the system dynamics and choice of oscillator frequencies to obtain the desired beam scanning angle. This paper also addresses the important issues of transient behavior of the phase shifts, location and existence of the equilibrium points and their corresponding regions of convergence.

Results of Monte Carlo simulation are presented and demonstrate the influence of randomness in the free running frequencies under different coupling strengths using a real design example of a six-element radar beam scanning array (Liao & York, 1994a). The influence of the coupling parameter on the detuning accuracy of oscillators is also examined using this practical design. This provides new insights that potentially enable researchers to study important problems such as quantifying the effect of oscillator manufacturing defects, and closed loop control for robust beam angle steering. This paper successfully merges two traditionally distinct fields of engineering knowledge: nonlinear control theory and antenna theory. It provides the radar community with an insight into the behavior of the coupled oscillators that did not exist before. It also provides the control community with an exciting potential application of control theory.

2. System dynamics

To predict the phase relationships in the coupled oscillator array, the dynamic analysis is required. The nonlinear differential equations describing instantaneous phase dynamics of one-dimensional coupled oscillator array were developed in York (1993) employing the time domain method. A single oscillator is modeled by RLC resonant circuits and coupled Van del Pol equations. The phase dynamic equations are

$$\dot{\theta}_i=\omega_i-\frac{\omega_i}{2Q}\sum_{k=1}^{N}\varepsilon_{ij}\frac{A_j}{A_i}\sin(\Phi_{ij}+\theta_i-\theta_j),\quad i=1,2,\ldots,N, \tag{1}$$

where ω_i, A_i, θ_i, Q, ε_{ij}, and Φ_{ij}, are the free-running frequency, instantaneous amplitude and phase of antenna, quality factor, coupling strength and coupling phase, respectively. It is noted that in this model, instead of considering a particular physical mechanism of couplings, the coupling network is described phenomenologically by

$$\kappa_{ij}\equiv\varepsilon_{ij}e^{-j\phi_{ij}}, \tag{2}$$

where κ_{ij} is the complex coupling coefficient. Nevertheless, several physical coupling mechanisms were successfully related to the complex number of κ_{ij} (York & Compton, 1993; York et al., 1994). Considering practical implementation, (an appropriate coupling network with constant coupling strengths and zero coupling phase, ($\phi_{ij}=0$), is generally chosen (Liao & York, 1993, 1994a,c; York et al., 1994). It is noted that the selection of zero coupling phase is just to simplify the analysis and design of the system and will not make the problem unrealistic. For example, it is demonstrated in Liao and York (1993) that the separation between the active elements determines the coupling phase and by setting a center-to-center spacing of $0.86\lambda_0$, the zero coupling phase could be obtained, where λ_0 is the signal wavelength in free space.

With the simplified coupling networks, (1) can be rewritten as

$$\dot{\theta}_i=\omega_i-\frac{\omega_i\varepsilon}{2Q}\sum_{\substack{j=i-1\\ j\neq i}}^{i+1}\frac{A_j}{A_i}\sin(\theta_i-\theta_j),\quad i=1,2,\ldots,N. \tag{3}$$

It should be noted that the phase distribution along the array steers the main beam angle, and the amplitude distribution, in most cases, determines the side lobe levels of the radiation pattern (Stutzman & Thiele, 1981). In this paper, only the oscillator array with a uniform amplitude distribution will be considered, such that $A_i=1$, $i=1,\ldots,N$. Then the dynamic equations are

$$\dot{\theta}_i=\omega_i-\omega_i\varepsilon'[\sin(\theta_i-\theta_{i-1})+\sin(\theta_i-\theta_{i+1})],\quad i=1,2,\ldots,N, \tag{4}$$

where $\varepsilon'=\varepsilon/2Q$.

The radiation pattern of a phased antenna array is steered at a desired direction by achieving a constant phase progression along the array (Stutzman & Thiele, 1981),

$$\Delta\theta=\frac{2\pi d}{\lambda}\sin\psi, \tag{5}$$

where ψ is the main beam direction from broadside, d is the spacing between adjacent elements of the array and λ_0 is the wavelength with respect to the synchronized frequency. Since the main beam scanning angle of a linear array is determined by the element-to-element phase shift $\Delta\theta_i$, the alternative state model of the system is to choose the phase shift between adjacent elements $\Delta\theta_i$ as the state variable. Replacing i with $i-1$, (4) becomes

$$\dot{\theta}_{i-1}=\omega_{i-1}-\omega_{i-1}\varepsilon'[\sin(\theta_{i-1}-\theta_{i-2})+\sin(\theta_{i-1}-\theta_i)],\quad i=2,\ldots,N. \tag{6}$$

Subtracting (4) by (6), obtains

$$\begin{aligned}\Delta\dot{\theta}_i=&\ \varepsilon'\omega_{i-1}\sin\Delta\theta_{i-1}-\varepsilon'(\omega_i+\omega_{i-1})\sin\Delta\theta_i\\&+\varepsilon'\omega_i\sin\Delta\theta_{i+1}+\omega_i-\omega_{i-1},\quad i=3,4,\ldots,N-1,\end{aligned} \tag{7}$$

where it is defined that

$$\Delta\theta_i=\theta_i-\theta_{i-1},$$

$$\Delta\theta_{i-1}=\theta_{i-1}-\theta_{i-2},$$

$$\Delta\theta_{i+1}=\theta_{i+1}-\theta_i. \tag{8}$$

Note that for $i=2$, $\Delta\theta_1=\theta_1-\theta_0$ does not exist, since there is no θ_0. The first nonlinear differential equation is

$$\Delta\dot{\theta}_2=-\varepsilon'(\omega_2+\omega_1)\sin\Delta\theta_2+\varepsilon'\omega_2\sin\Delta\theta_3+\omega_2-\omega_1. \tag{9}$$

Similarly, $\Delta\theta_{N+1}$ does not exist either. The last equation is derived as

$$\Delta\dot{\theta}_N=\varepsilon'\omega_{N-1}\sin\Delta\theta_{N-1}-\varepsilon'(\omega_N+\omega_{N-1})\sin\Delta\theta_N+\omega_N-\omega_{N-1}. \tag{10}$$

Finally the state model of choosing the phase shift $\Delta\theta$ as the state variable is given as

$$\begin{aligned}\Delta\dot{\theta}_2&=-\varepsilon'(\omega_2+\omega_1)\sin\Delta\theta_2+\varepsilon'\omega_2\sin\Delta\theta_3+\omega_2-\omega_1,\\\Delta\dot{\theta}_i&=\varepsilon'\omega_{i-1}\sin\Delta\theta_{i-1}-\varepsilon'(\omega_i+\omega_{i-1})\sin\Delta\theta_i+\varepsilon'\omega_i\sin\Delta\theta_{i+1}+\omega_i-\omega_{i-1},\\&\ \vdots\\\Delta\dot{\theta}_N&=\varepsilon'\omega_{N-1}\sin\Delta\theta_{N-1}-\varepsilon'(\omega_N+\omega_{N-1})\sin\Delta\theta_N+\omega_N-\omega_{N-1}.\end{aligned} \tag{11}$$

Its vector form will be simply denoted as

$$\boldsymbol{\Delta\dot{\theta}}=f(\boldsymbol{\Delta\theta}). \tag{12}$$

This state model uses the relative phase shift, $\Delta\theta$, as the state variable instead of the instantaneous phase. Compared with the state model involving the instantaneous phase, this model