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# Receding-horizon optimal control of the current profile evolution during the ramp-up phase of a tokamak discharge

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ABSTRACT

The control of the toroidal current density spatial profile in tokamak plasmas will be absolutely critical in future commercial-grade reactors to enable high fusion gain, non-inductive sustainment of the plasma current for steady-state operation, and magnetohydrodynamic (MHD) instability-free performance. The evolution in time of the current profile is related to the evolution of the poloidal magnetic flux, which is modeled in normalized cylindrical coordinates using a partial differential equation (PDE) usually referred to as the magnetic flux diffusion equation. The control objective during the ramp-up phase is to drive an arbitrary initial profile to approximately match, in a short time windows during the early flattop phase, a predefined target profile that will be maintained during the subsequent phases of the discharge. Thus, such a matching problem can be treated as an optimal control problem for a PDE system. A distinctive characteristic of the current profile control problem in tokamaks is that it admits interior, boundary and diffusivity actuation. A receding-horizon control scheme is proposed in this work to exploit this unique characteristic and to solve the associated open-loop finite-time optimal control problem using different optimization techniques. The efficiency of the proposed scheme is shown in simulations.



## 1. Introduction

The need for new sources of energy is expected to become a critical problem within the next few decades. It is a fact that fossil fuel energy is becoming more expensive and polluting. Nuclear fission and fusion are candidate sources of energy with sufficient energy density to supply the increasing world population with its steadily increasing energy demands. In both fission and fusion reactions the total masses after the reaction are less than those before. The "lost" mass appears as energy, with the amount given by the famous Einstein formula, $E=(M_r-M_p)c^2$, where $E$ is the energy, $M_r$ is the mass of the reactant nuclei, $M_p$ is the mass of the product nuclei, and $c$ is the speed of light. In a fission reaction, a heavy nucleus splits apart into smaller nuclei. Fission is a mature technology powering present nuclear power reactors. In a fusion reaction, on the contrary, two light nuclei (deuterium and tritium (two isotopes of hydrogen)) stick together to form a heavier nucleus (helium) plus an energetic neutron. Like fission, fusion produces no air pollution or greenhouse gases, since the reaction product is helium. Unlike fission, fusion poses no risk of nuclear accident, generation of high-level nuclear waste, and production of material for nuclear weapons. In addition, there is an abundant fuel supply. Deuterium, may be readily extracted from ordinary water, which is available to all nations. Tritium does not occur naturally but would be produced from lithium (through a nuclear reaction that makes use of the neutron resulting from the D–T fusion process), which is available from land deposits or from sea water which contain thousands of years' supply. The world-wide availability of these materials would thus eliminate international tensions caused by imbalance in fuel supply.

Since nuclei carry positive charges, they normally repel one another when trying to fuse. To overcome the Coulomb barrier, the kinetic energy of the nuclei must be increased by heating. The fusion process requires extremely high temperatures (50–200 million Kelvin), at which the hydrogen gas ionizes and becomes a plasma. Within a plasma, electrons are free to move independently of the nucleus and the gas is essentially a sea of charged particles, which conduct electricity and interact with magnetic fields. One of the most promising approaches to fusion is indeed the magnetic confinement concept, which exploits these properties of the plasma. Strong magnetic fields act like a magnetic bottle to hold the ionized (charged) nuclei together and away from the vessel wall as they are heated to fusion temperatures. A Russian design in the shape of a torus, called

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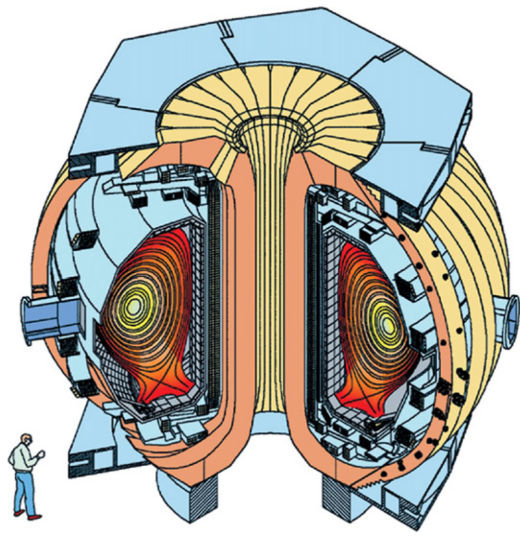

**Fig. 1.** Scheme of the DIII-D Tokamak. The toroidal field (TF) coils (creamy yellow) are wrapped "poloidally" around the torus (the short way, going through the center hole), while the poloidal field (PF) coils (light blue) are wrapped "toroidally" (the long way) around the torus. Current flowing in these conducting coils produces the helical magnetic field that confines the plasma. The plasma contained within the device is represented by a set of nested contours of constant magnetic flux. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

tokamak (Fig. 1), has proved particularly well suited for containing a fusion reaction. A more in-depth introduction to fusion can be found in Leuer (1995), Pironti and Walker (2005), Walker et al. (2006) and Schuster and Ariola (2006), in which considerable effort was made to describe the current problems of tokamak plasma control at a level that is accessible to engineers, mathematicians, and non-plasma physicists.

In a tokamak (Fig. 1), the magnetic field lines twist their way around the torus to form a helical structure. The toroidal magnetic field component is produced by the so-called "toroidal field" (TF) coils. Addition of a poloidal magnetic field component, generated by the toroidal plasma current and the "poloidal field" (PF) coils, is necessary for the existence of a magnetohydrodynamic (MHD) equilibrium (Freidberg, 1987). It is possible to use the poloidal component of the helicoidal magnetic lines to define nested toroidal surfaces corresponding to constant values of the poloidal magnetic flux. The poloidal flux $\psi$ at a point $P$ is the total flux through the surface $S$ bounded by the toroidal ring passing through $P$, i.e., $\psi = \int B_{pol}\, dS$. The dynamics of the poloidal magnetic flux is governed by a parabolic partial differential equation (PDE) usually referred to as the magnetic flux diffusion equation. The shape of the poloidal magnetic flux profile has a direct effect on the current density profile since they are related by spatial derivative operations.

The need to optimize the tokamak concept for the design of an economical, possibly steady state, fusion power plant have motivated extensive international research aimed at finding the so-called "advanced tokamak (AT) operation scenarios" (Taylor, 1997). In a large number of machines, experiments have demonstrated the existence of such regimes characterized by a high confinement state with improved MHD stability, which yields a strong increase of the plasma performance quantified by the normalized energy confinement time and plasma pressure. In such conditions a dominant fraction of the plasma current is self-generated by the neo-classical bootstrap mechanism, which alleviates the requirement on externally driven current and enables steady-state operation. This highly confined state is achieved to a large extent by the generation of a so-called "internal transport barrier" (ITB) (Connor et al., 2004), a region where the plasma turbulence (and therefore the plasma transport) is almost suppressed. Many studies have shown the key influence of the current density profile on triggering the ITBs (Challis, 2004). This provides a strong motivation for the control of the current density profile in real time.

Recent experiments in different devices around the world (JET, (Laborde et al., 2005; Moreau et al., 2003, 2008), DIII-D (Ferron et al., 2006), JT-60U (Suzuki et al., 2005), Tore Supra (Barana, Mazon, Laborde, & Turco, 2007; Wijnands et al., 1997) have demonstrated significant progress in achieving profile control. At JET, different current and temperature gradient target profiles have been reached and sustained for several seconds during the flattop current phase. The control schemes rely on the experimental identification of linearized static (Laborde et al., 2005; Moreau et al., 2003) and dynamic (Moreau et al., 2008) response models, using lower hybrid current drive (LHCD), ion cyclotron resonance heating (ICRH) and neutral beam injection (NBI) as actuators. The controllers, which finally reduce to proportional-integral regulators incorporating information of the identified response of the system and exploiting the different time scales of kinetic and magnetic variables, have been proved effective in experiments. Experiments at DIII-D (Ferron et al., 2006) focus on creating the desired current density profile during the plasma current ramp-up and early flattop phases with the aim of maintaining this target profile during the subsequent phases of the discharge. Since the actuators that are used to achieve the desired target profile are constrained, experiments have shown that some of the desirable target profiles may not be achieved for all arbitrary initial conditions. Therefore, a perfect matching of the desirable target profile may not be physically possible. In practice, the objective is to achieve the best possible approximate matching in a short time windows $[T_1, T_2]$ during the early flattop phase of the total plasma current pulse, as shown in Fig. 2. Thus, such a matching problem can be treated as a finite-time optimal control problem for a parabolic PDE system.

The control of the current density profile in tokamak plasmas is unique in the sense that it admits actuation not only through interior control (see, e.g., Christofides, 2001 and references therein) and boundary control (see, e.g., Krstic & Smyshlyaev, 2008 and references therein) but also through what is named

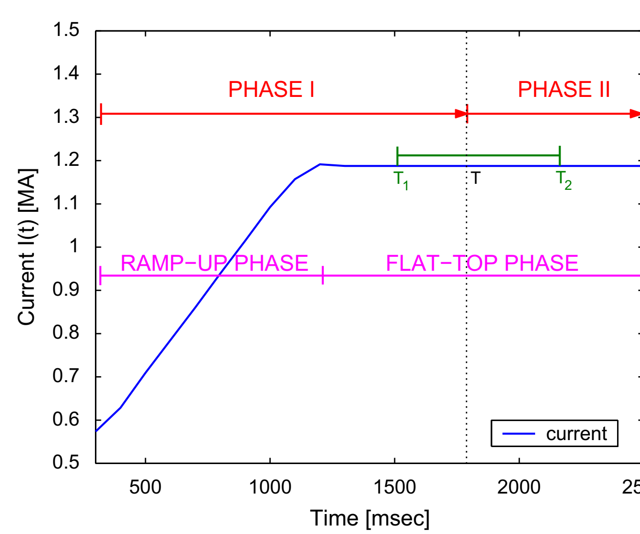


**Fig. 2.** The total plasma current evolution can be roughly divided into two phases: the ramp-up phase and the flat-top phase. The control problem focuses on phase I that includes the ramp-up phase and the first part of the flat-top phase. The control goal is to drive the current profile from some initial arbitrary condition to a predefined target profile at some time $T$ between the time window $[T_1, T_2]$, which is in the flat-top phase.