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Switching fuzzy tracking control for mobile robots under curvature constraints

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ABSTRACT

In this paper a switching fuzzy logic controller for mobile robots with a bounded curvature constraint is presented. The controller tracks piece-wise linear paths, which are an approximation of the feasible smooth reference path. The controller is constructed through the use of a map, which transforms the problem to a simpler one; namely the tracking of straight lines. This allows the use of an existing fuzzy tracker deployed in a previous work, and its simplification leading to a 70% rule reduction. Simulation results and a comparison analysis with existing trackers are also presented along with some stability considerations on the impulsive error dynamics which emerge.

1. Introduction

Path tracking is an essential part of the navigation process of every mobile robot that follows the *deliberative control* paradigm. In this setting, the robot uses state and environment information in order to plan ahead its movements. On the contrary, *reactive control* systems employ a tight coupling between perception and motor action leading to more timely responses, but lacking the predictive capabilities of the former.

In a typical case, the path is provided by a path planner that calculates a feasible path between the start and goal positions. The planner can take into account special restrictions such as the distance from obstacles, kinematic and dynamic constraints of the robotic system, total travel length, etc. The path is then presented to the path tracker that performs the actual movement. The aim of the tracker is to navigate the robot on the path, accounting for disturbances in the moving phase, e.g., wheel slippage, uneven terrain, localization uncertainty, etc. The study of path tracking has produced a vast amount of research literature ranging from classical control approaches (Altafini, 1999; Kamga & Rachid, 1997; Kanayama & Fahroo, 1997), to nonlinear control methodologies (Altafini, 2002; Egerstedt, Hu, & Stotsky, 1998; Koh & Cho, 1994; Samson, 1995; Wit, Crane, & Armstrong, 2004) to intelligent control strategies (Abdessemed, Benmahammed, & Monacelli, 2004; Antonelli, Chiaverini, & Fusco, 2007; Baltes & Otte, 1999; Cao & Hall, 1998; Deliparaschos, Moustris, & Tzafestas, 2007; El Hajjaji & Bentalba, 2003; Lee, Lam, Leung, & Tam, 2003; Liu & Lewis, 1994; Maalouf, Saad, & Saliah, 2006; Moustris & Tzafestas, 2005; Rodríguez-Castaño, Heredia, & Ollero, 2000; Sanchez, Ollero, & Heredia, 1997; Xinxin, Kezhong, Muhe, & Bo, 1998). Of course, boundaries often blend since various approaches are used simultaneously.

Fuzzy logic path trackers have been used by several researchers (Abdessemed et al., 2004; Antonelli et al., 2007; Baltes & Otte, 1999; Cao & Hall, 1998; Deliparaschos et al., 2007; El Hajjaji & Bentalba, 2003; Jiangzhou, Sekhavat, & Laugier, 1999; Lee et al., 2003; Liu & Lewis, 1994; Moustris & Tzafestas, 2005; Ollero, Garcia-Cerezo, Martinez, & Mandow, 1997; Raimondi & Ciancimino, 2008; Rodríguez-Castaño et al., 2000; Sanchez et al., 1997) since fuzzy logic provides a more intuitive way of analyzing and formulating the control actions, which bypasses most of the mathematical load needed to tackle such a highly nonlinear control problem. Furthermore, the fuzzy controller, that can be less complex in its implementation, is inherently robust to noise and parameter uncertainties.

In most tracking controllers, the general strategy is to actually track a point on the reference path. With respect to that point error signals can be defined and the problem can be reformulated as a stabilization problem of the *error dynamics*. Specifically, let $x_r(t) \in \mathbb{R}^n$ be the reference path, $x(t) \in \mathbb{R}^n$ be the robot's state vector and $e(t) \in \mathbb{R}^k$ be the error vector. Furthermore, if $S : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a *selection* function, and $d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^k$, $d(0,0)=0$ is an appropriate *distance* function one can write

$$x_r^p(t) = S(x(t), x_r(t)), \quad x_r^p \in x_r \tag{1}$$

and,

$$e = d(x(t), x_r^p(t)) \tag{2}$$

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The error dynamics can then be defined by

$$\dot{e} = h(e,u,t) \tag{3}$$

where $u \in \mathbb{R}^m$ is the control input and h is a function derived from Eq. (2) and the system function. The *selection* function incorporates the mechanism for selecting the *tracking point* $x_r^p(t)$ on the reference path. For example, the function S can express the minimum Euclidean distance of the robot to the reference path, in which case the tracking point is the closest point of the path to the robot. It is noted that the selection function need not be differentiable or even continuous.

In the above formulation of the tracking problem, the goal is to find a control law of the form $u = u(e,t)$, which assures that the origin of Eq. (3) is locally (or globally) uniformly asymptotically stable. Although asymptotic or even exponential stability is generally desired, in many practical implementations this cannot be achieved. This situation arises when the reference path is *not* an actual feasible trajectory of the mobile robot. In practice, many researchers consider piece-wise linear reference paths that present derivative discontinuities at the vertex points. Such paths are easier to handle computationally and are less complex in control implementations. They can be defined by a collection of points and can approximate an actual solution. Essentially, these paths are derived from the sampling of a smooth reference path and express its polygonal approximation. Clearly, for mobile robots that include bounding constraints on their curvature (such as the Dubins car or the car-like robot) these paths are not actual solutions of the system equations, but rather describe piece-wise solutions. Consequently, the error dynamics presents impulsive effects that perturb the error from the equilibrium point. More specifically, the impulse effect is presented in the *heading error*, which describes the difference of the robot's heading to the tracking point's heading. When the tracking point changes branch, its heading "jumps" thus inserting an impulse on the error signal (Fig. 1).

The resulting error dynamics is a switched impulsive system with state dependent switching surfaces i.e. the reset map $\boldsymbol{M}$ is a subset of the system's state-space and is time invariant. The system equations are of the form

$$\begin{aligned} &\dot{x}(t) = f(x(t)), \quad x(t) \notin \boldsymbol{M} \\ &x(t^+) = I(x(t)), \quad x(t) \in \boldsymbol{M} \end{aligned} \tag{4}$$

where $I : \mathbb{R}^n \to \mathbb{R}^n$ is the *reset law*. The reset map $\boldsymbol{M}$ is defined by the geometry of the reference path *and* the actual selection function used. Eq. (4) can be reformulated involving *mobile time instances* of impulsive jumps. If the system hits the switching surfaces at the time instances τ_i, the discrete set $E = \{\tau_1, \tau_2 \ldots : \tau_1 < \tau_2 < \ldots\} \subset \mathbb{R}^+$ denotes the instances the jumps occur, and Eq. (4) becomes

$$\begin{aligned} &\dot{x}(t) = f(x(t)), \quad t \notin E \\ &x(t^+) = I_k(x(t)), \quad t \in E \end{aligned} \tag{5}$$

Notice that the time instances depend on the actual motion of the system, i.e. the set E is not predetermined.

Another repercussion of the introduction of impulsive effects in the tracking control is that, with respect to the Dubins Car model, the zero set of the error dynamics is not an equilibrium point. An insight of this behavior can be grasped by inspection of Fig. 1. If the system is allowed to perfectly track each edge, thus driving the error solution to zero, when the robot changes branch the impulse will perturb the heading error. Thus the origin in not an actual invariant set of the system and consequently cannot be an equilibrium point (see Shen & Jing, 2006 for some basic definitions on impulsive systems). Quantitatively this can be deduced from the fact, as will be shown later, that $f(0) = 0$ but $I_k(0) \neq 0$.

This paper has a twofold purpose. The first is to present a switching fuzzy tracking controller for the Dubins Car, tracking polygonal paths. The controller is a Takagi-Sugeno zero-order type FLC with triangular membership functions and an overlap of two. The tracking control of such paths is rather important since many tracking applications resort in practice to such path formulations, although the effect of the polygonization is often overlooked since the error is kept small. The second purpose however, is the presentation of the actual methodology used that resulted to this controller. Note the word "resulted", since the controller has not been designed *ab initio*, but rather is a simplification of an existing fuzzy tracker, which the authors have used in previous experiments (Deliparaschos et al., 2007). The prime methodology used is the application of the strip-wise affine map.

The strip-wise affine map is a piece-wise linear homeomorphism that maps a planar polygonal path in the so-called *physical domain* (the actual plane where the robot moves) to a straight line in the *canonical domain*. In order to impose bijectivity, the path must be a strictly monotone polygonal chain. This mapping enables the reduction of path tracking to straight line tracking, something that simplifies the design of tracking controllers for mobile robots. Furthermore, the transformation also preserves the system equations, meaning that it maps a car-like robot in the physical domain, to a car-like robot in the canonical domain. This enables one to use existing tracking controllers for path tracking, albeit simplified ones in order to track straight lines.

The simplification consists of reducing the fuzzy path tracker to a straight line tracking controller, i.e. tailoring it to tracking only straight lines, in contrast to arbitrary polygonal paths. This is made feasible due to the very nature of the strip-wise affine map

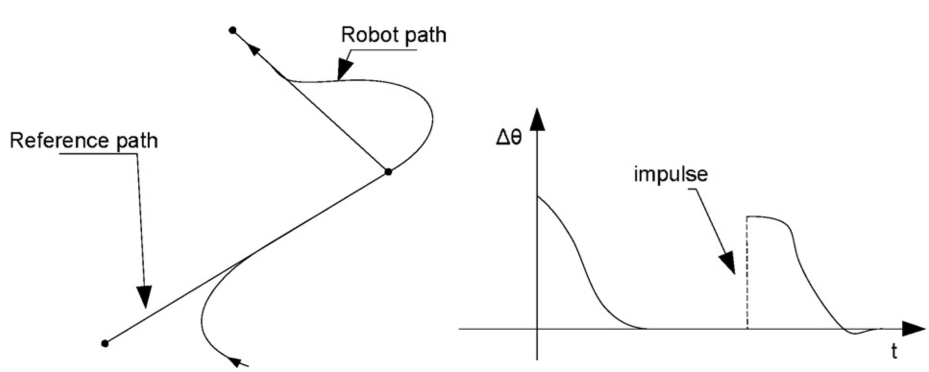

Fig. 1. Impulsive effect on piece-wise linear paths.